

Honors Algebra 2 Review for Final

Part I: No Calculator

1. A polynomial  $f$  and one zero are given. Find the other zeros of  $f$ .

a)  $f(x) = x^3 + 2x^2 - 20x + 24; -6$

$$\begin{array}{r|rrrr} -6 & 1 & 2 & -20 & 24 \\ & & -6 & 24 & -24 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

zeros:  
 $x = -6, 2$

$$f(x) = (x^2 - 4x + 4)(x + 6)$$

$$= (x - 2)(x - 2)(x + 6)$$

b)  $f(x) = 15x^3 - 119x^2 - 10x + 16; 8$

$$\begin{array}{r|rrrr} 8 & 15 & -119 & -10 & 16 \\ & & 120 & 8 & -16 \\ \hline & 15 & 1 & -2 & 0 \end{array}$$

$$f(x) = (15x^2 + x - 2)(x - 8)$$

$$= (3x - 1)(5x + 2)(x - 8)$$

2. (Calc Active) Find the local max and mins of the following equations.

a)  $f(x) = x^4 - 4x^3 + 2x^2 + x + 4$

b)  $f(x) = x^3 + 11x^2 + 35x + 32$

zeros  
 $x = 8, 1/3, -2/5$

min

$(-0.164, 3.908)$  and  $(2.574, -4.494)$

min  $(-2.\bar{3}, -2.482)$

max

$(0.5907, 4.5859)$

max  $(-5, 7)$

3. Find all possible rational zeros of the function

$f(x) = x^8 - 17x^4 + 16$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$   
 $\pm 1$

$= \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

4. Determine how many possible positive, negative, and imaginary zeros

5 zeros  $f(x) = x^5 - 3x^4 - x^3 + 3x^2 - 6x + 18$  may have.

$f(-x) = (-x)^5 - 3(-x)^4 - (-x)^3 + 3(-x)^2 - 6(-x) + 18$

4 pos 1 neg

2 pos 1 neg 2 imaginary

0 pos 1 neg 4 imaginary

$= -x^5 - 3x^4 + x^3 + 3x^2 + 6x + 18$

5. Graph the following function:

$$f(x) = (x-6)(x+1)^2(x-2)$$

cross    bounce    cross

Degree: 4

Sign of the Leading Coefficient: pos

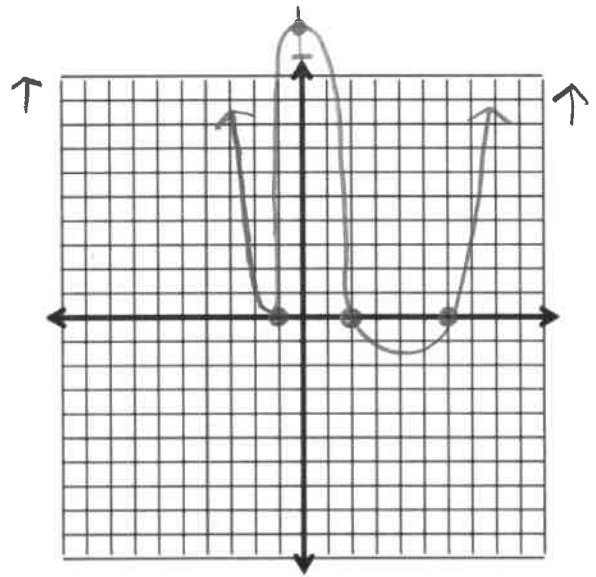
Y-intercept: (0, 12)

Zeros: (6, 0) (-1, 0) (2, 0)

Multiplicity of zeros: 1, 2, 1

Domain:  $(-\infty, \infty)$

~~Range:~~ don't know for certain w/o calc



6. Simplify using properties of exponents

a.  $x^m \cdot x^n = x^{m+n}$

c.  $(x^m)^n = x^{mn}$

e.  $(xy)^n = x^n y^n$

b.  $\frac{x^m}{x^n} = x^{m-n}$

d.  $x^{-n} = \frac{1}{x^n}$

f.  $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

7. Simplify

a.  $11^{\frac{1}{3}}$  (write as a radical)

$$\sqrt[3]{11}$$

b.  $\sqrt[3]{5^2}$  (write using fractional exponents)

$$5^{\frac{2}{3}}$$

8. Evaluate

a.  $512^{\frac{2}{3}}$

$$\sqrt[3]{512}^2$$

$$\sqrt[3]{2^9}^2$$

$$(2^3)^2$$

$$(8)^2$$

$$\boxed{64}$$

b.  $(-243)^{\frac{1}{5}}$

$$\sqrt[5]{-243}$$

$$\sqrt[5]{-3^5}$$

$$\boxed{-3}$$

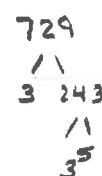
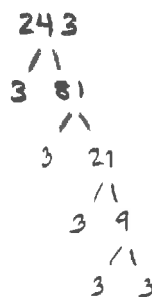
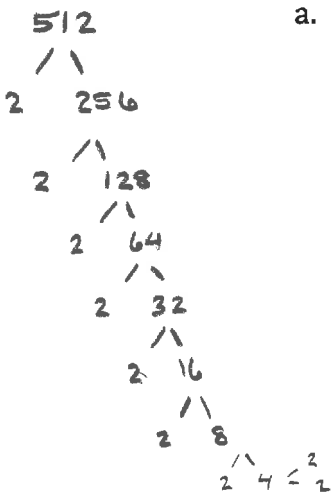
c.  $729^{-\frac{2}{3}}$

$$\frac{1}{\sqrt[3]{729}^2}$$

$$\frac{1}{\sqrt[3]{3^6}^2}$$

$$\frac{1}{(3^2)^2} = \frac{1}{(9)^2}$$

$$= \boxed{\frac{1}{81}}$$



d.  $81^{\frac{1}{3 \cdot 4}}$   
 $= \sqrt[4]{81^3} = 3^3$   
 $= \sqrt[4]{3^4} = 3$  27

$\begin{matrix} 324 \\ / \quad \backslash \\ 2 \quad 162 \\ / \quad \backslash \\ 2 \quad 81 \\ / \quad \backslash \\ 3 \quad 27 \\ / \quad \backslash \\ 3 \quad 9 \\ / \quad \backslash \\ 3 \quad 3 \\ / \quad \backslash \\ 3 \quad 1 \end{matrix}$

$\begin{matrix} 361 \\ / \quad \backslash \\ 19 \quad 19 \end{matrix}$

e.  $(\frac{361}{324})^{-1/2}$   
 $= \frac{\sqrt{324}}{\sqrt{361}}$   
 $= \frac{2 \cdot 3^2}{19}$   $\frac{18}{19}$

f.  $(-64)^{4/3}$   
 $= \sqrt[3]{-64^4}$   
 $= \sqrt[3]{-4^3^4}$   
 $= (-4)^4$   
256

9. Evaluate

a.  $\log_{1/81} 3 = -1/4$   $-1/4$   
 $(\frac{1}{81})^x = 3$   
 $(3^{-4})^x = 3$   
 $-4x = 1$   $x = -1/4$

f.  $\log_{16} 64 = 3/2$   $3/2$   
 $16^x = 64$   
 $4^{2x} = 4^3$   
 $2x = 3$   $x = 3/2$

k.  $e^{\ln 7} = 7$  7

b.  $\log_4 1 = 0$  0

g.  $\log_4 \frac{1}{\sqrt[3]{16}} = -2/3$   $-2/3$   
 $4^x = \frac{1}{\sqrt[3]{16}}$   
 $4^x = 16^{-1/3}$   $x = -2/3$

l.  $\ln 1 = 0$  0

c.  $\ln e^4 = 4$  4

h.  $\ln e = 1$  1

m.  $\log_3 3^4 - \log_8 8^4$   
 $4 - 4$   
0

d.  $\log \sqrt[5]{10} = 1/5$   $1/5$   
 $10^x = \sqrt[5]{10}$   
 $10^x = 10^{1/5}$   
 $x = 1/5$

i.  $\log 1000 = 3$  3  
 $10^x = 1000$   
 $10^x = 10^3$   
 $x = 3$

n.  $\log_2 64 - 7 \log_7 7$   
 $6 - 7$   
-1

e.  $5^{\log_5 12} = 12$  12

j.  $\log_9 3 = 1/2$   $1/2$   
 $9^x = 3$   
 $x = 1/2$

o.  $\log_3 \frac{1}{81} + \log_4 64$   
 $3^x = \frac{1}{81}$   $4^x = 64$   
 $3^x = 3^{-4}$   $4^x = 4^3$   
 $-4 + 3$   
-1

p.  $e^{\ln 4}$   
 $= 4$

r.  $\ln e^9$   
 $= 9$

t.  $10^{\log 0.2}$   
 $= 0.2$

q.  $e^{2 \ln 3}$   
 $e^{\ln 3^2}$   
 $= 3^2$   
 $= 9$

s.  $5 \ln e^3$   
 $= 5(3)$   
 $= 15$

u.  $\log_{25} 125 = 3/2$   
 $25^x = 125$   
 $5^{2x} = 5^3$   
 $2x = 3$   
 $x = 3/2$

10. Solve for  $x$ . Check for extraneous roots.

a.  $4(x-2)^3 = 32$

$$(x-2)^3 = 8$$

$$x-2 = \sqrt[3]{8}$$

$$x = 2 + 2$$

$$x = 4$$

$$\begin{array}{r} 256 \\ / \quad \backslash \\ 2 \quad 128 \\ / \quad \backslash \\ 2 \quad 64 \\ / \quad \backslash \\ 2 \quad 32 \\ / \quad \backslash \\ 2 \quad 16 \\ / \quad \backslash \\ 2 \quad 8 \\ / \quad \backslash \\ 2 \quad 4 \\ / \quad \backslash \\ 2 \quad 2 \\ / \quad \backslash \\ 2 \quad 1 \end{array}$$

c.  $(x-5)^4 = 256$

$$x-5 = \sqrt[4]{256}$$

$$x = 5 + \sqrt[4]{2^8}$$

$$= 5 + 2^2$$

$$= 9$$

b.  $x^6 + 50 = 779$

$$x^6 = 729$$

$$x = \sqrt[6]{729}$$

$$x = \sqrt[6]{3^6}$$

$$x = 3$$

$$\begin{array}{r} 729 \\ / \quad \backslash \\ 3 \quad 243 \\ / \quad \backslash \\ 3 \quad 81 \\ / \quad \backslash \\ 3 \quad 27 \\ / \quad \backslash \\ 3 \quad 9 \\ / \quad \backslash \\ 3 \quad 3 \\ / \quad \backslash \\ 3 \quad 1 \end{array}$$

d.  $m^{3/4} = 27$

$$m = 27^{4/3}$$

$$= \sqrt[3]{27^4}$$

$$= 3^4$$

$$= 81$$

$$\begin{aligned}
 \text{e. } \sqrt{2x-7} &= x-3 \\
 2x-7 &= (x-3)^2 \\
 2x-7 &= x^2-6x+9 \\
 0 &= x^2-8x+16 \\
 0 &= (x-4)^2 \\
 &\boxed{x=4}
 \end{aligned}$$

$$\begin{aligned}
 \text{f. } 3(x-5)^{2/3}-6 &= 18 \\
 3(x-5)^{2/3} &= 24 \\
 (x-5)^{2/3} &= 8 \\
 x-5 &= 8^{3/2}
 \end{aligned}$$

$$x = 5 \pm \sqrt{8^3} = 5 \pm \sqrt{2^3 \cdot 2^3}$$

$$= 5 \pm \sqrt{2^6} = 5 \pm 2^3 \sqrt{2} = \boxed{5 \pm 16\sqrt{2}}$$

11. Simplify.

$$\begin{aligned}
 \text{a. } \frac{x^3-27}{x^2-9} &= \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)} \\
 &= \boxed{\frac{x^2+3x+9}{x+3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \frac{x^2+11x+28}{2x^2+8x} &= \frac{(x+7)(x+4)}{2x(x+4)} \\
 &= \boxed{\frac{x+7}{2x}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } \frac{27e^7}{9e^3} &= \boxed{3e^4}
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } \sqrt[4]{8} \cdot \sqrt[4]{8} &= \sqrt[4]{64} \\
 &= \sqrt[4]{2^6} \\
 &= \boxed{2\sqrt[4]{4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } 10\sqrt[5]{4} - 18\sqrt[5]{4} &= \boxed{-8\sqrt[5]{4}}
 \end{aligned}$$

$$\text{f. } \sqrt[3]{81x^7}$$

$$\begin{aligned}
 &\sqrt[3]{3^4 x^7} \\
 &\boxed{3x^2 \sqrt[3]{3x}}
 \end{aligned}$$

$$\begin{aligned}
 \text{g. } \sqrt{3x+7} &= x+1 \\
 3x+7 &= x^2+2x+1 \\
 0 &= x^2-x-6 \\
 0 &= (x-3)(x+2) \\
 x &= 3, -2 \\
 &\boxed{x=3}
 \end{aligned}$$

$$\begin{aligned}
 \text{h. } \sqrt{x-4} &= \sqrt{2x} \\
 x-4 &= 2x \\
 -4 &= x \\
 &\boxed{\text{no solution}}
 \end{aligned}$$

$$\text{g. } \sqrt[5]{64x^7y^{20}z^{23}}$$

$$= \sqrt[5]{2^5 \cdot 2^5 x^5 y^{20} z^{20} z^3}$$

$$= \boxed{2xy^4z^4 \sqrt[5]{2x^2z^3}}$$

$$\text{h. } (-48)^{1/3} + (750)^{1/3}$$

$$\sqrt[3]{-2^3 \cdot 6} + \sqrt[3]{5^3 \cdot 6}$$

$$-2\sqrt[3]{6} + 5\sqrt[3]{6}$$

$$\boxed{3\sqrt[3]{6}}$$

$$\text{i. } 2\sqrt{3} + 7\sqrt{3}$$

$$\boxed{9\sqrt{3}}$$

$$\begin{array}{r}
 48 \\
 2 \overline{) 48} \\
 \underline{40} \phantom{0} \\
 80 \\
 \underline{80} \\
 0
 \end{array}$$

$$\begin{array}{r}
 750 \\
 2 \overline{) 750} \\
 \underline{500} \phantom{0} \\
 250 \\
 \underline{250} \\
 0
 \end{array}$$

$$\begin{array}{r}
 75 \\
 5 \overline{) 75} \\
 \underline{75} \\
 0
 \end{array}$$

$$\begin{array}{r}
 3 \ 25 \\
 3 \overline{) 75} \\
 \underline{60} \phantom{0} \\
 150 \\
 \underline{150} \\
 0
 \end{array}$$

j.  $9\sqrt{2} - 5\sqrt{2}$

$$\boxed{4\sqrt{2}}$$

k.  $\sqrt[3]{7} + 5\sqrt[3]{56}$

$$\sqrt[3]{7} + 5\sqrt[3]{8 \cdot 7}$$

$$\sqrt[3]{7} + 5 \cdot 2 \sqrt[3]{7}$$

$$\boxed{11\sqrt[3]{7}}$$

l.  $17\sqrt[5]{4} - 2\sqrt[5]{128}$

$$17\sqrt[5]{4} - 2\sqrt[5]{2^7}$$

$$17\sqrt[5]{4} - 2 \cdot 2 \sqrt[5]{2^2}$$

$$17\sqrt[5]{4} - 4\sqrt[5]{4}$$

$$\boxed{13\sqrt[5]{4}}$$

12. Rationalize:

a)  $\frac{3}{7-\sqrt{5}} \cdot \frac{7+\sqrt{5}}{7+\sqrt{5}}$

$$\frac{21 + 3\sqrt{5}}{49 - 5}$$

$$\boxed{\frac{21 + 3\sqrt{5}}{44}}$$

b)  $\frac{4}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$$\frac{12 - 4\sqrt{2}}{9 - 2}$$

$$\boxed{\frac{12 - 4\sqrt{2}}{7}}$$

c)  $\frac{2}{\sqrt[3]{32}}$

$$\frac{2}{\sqrt[3]{2^5}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$$

$$\frac{2\sqrt[3]{2}}{\sqrt[3]{2^6}}$$

$$\frac{2\sqrt[3]{2}}{2^2}$$

$$\boxed{\frac{\sqrt[3]{2}}{2}}$$

d)  $\frac{5}{\sqrt[3]{9}}$

$$\frac{5}{\sqrt[3]{3^2}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$$

$$\boxed{\frac{5\sqrt[3]{3}}{3}}$$

13. Let  $f(x) = x^{11} - 4$ . Find  $f^{-1}(x)$  and verify your answer.

$$x = y^{11} - 4$$

$$x + 4 = y^{11}$$

$$\boxed{\sqrt[11]{x+4} = f^{-1}(x)}$$

verify:

$$f(f^{-1}(x)) = (\sqrt[11]{x+4})^{11} - 4$$

$$= x + 4 - 4$$

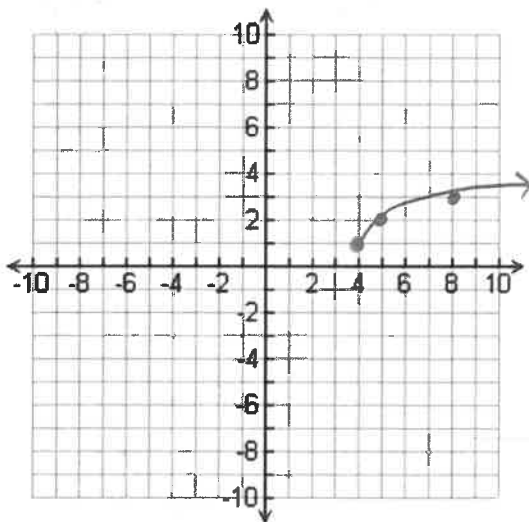
$$= x \quad \checkmark$$

$$f^{-1}(f(x)) = \sqrt[11]{(x^{11} - 4) + 4}$$

$$= \sqrt[11]{x^{11}}$$

$$= x \quad \checkmark$$

14. Graph the function  $f(x) = \sqrt{x-4} + 1$ . Find the domain, range, and state ALL transformations.  $\rightarrow 4 \uparrow 1$



Domain:  $[4, \infty)$

Range:  $[1, \infty)$

Transformations:

right 4

up 1

15. Solve:

a)  $4^{2x+4} = 16^{3x-6}$     b)  $(0.25)^{x+8} = (0.5)^{x^2+1}$     c)  $\log_3(1-3x)+1=5$     d)  $\log_3(x+2) - \log_3(x-1) = 2$

$4^{2x+4} = (4^2)^{3x-6}$

$2x+4 = 6x-6$

$16 = 4x$

$x = 4$

$(\frac{1}{4})^{x+8} = (\frac{1}{2})^{x^2+1}$

$[(\frac{1}{2})^2]^{x+8} = (\frac{1}{2})^{x^2+1}$

$2x+16 = x^2+1$

$0 = x^2 - 2x - 15$

$0 = (x-5)(x+3)$

$x = 5, -3$

$\log_3(1-3x) = 4$

$3^4 = 1-3x$

$81 = 1-3x$

$80 = -3x$

$-\frac{80}{3} = x$

$\log_3 \frac{x+2}{x-1} = 2$

$3^2 = \frac{x+2}{x-1}$

$9 = \frac{x+2}{x-1}$

$9x-9 = x+2$

$8x = 11$

$x = \frac{11}{8}$

16. Perform the indicated operation. Write your answer in simplest form.

a)  $\frac{x^3 - 8}{3x^2 - 10x + 8} \cdot \frac{6x^2 - 8x}{2x^3 + 4x^2 + 8x} \cdot 2x(x + 2x + 4)$

$$\frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(3x-4)}(x-2)} \cdot \frac{2x\cancel{(3x-4)}}{2x(x+2x+4)}$$

$$= 1$$

b)  $\frac{x^3 + 4x}{2x - 1} \div \frac{x^4 - 16}{x^2 - 4x - 12}$

$$\frac{x(x^2 + 4)}{2x - 1} \cdot \frac{(x - 6)(x + 2)}{(x^2 - 4)(x^2 + 4)}$$

$$\frac{x}{2x - 1} \cdot \frac{(x - 6)\cancel{(x - 2)}}{\cancel{(x - 2)}(x + 2)} = \frac{x(x - 6)}{(2x - 1)(x + 2)}$$

c)  $\frac{16x}{4x - 8} \div \frac{x^2}{x^2 - 4} \cdot \frac{x + 6}{8}$

$$\frac{\cancel{16}x}{4(x - 2)} \cdot \frac{(x - 2)(x + 2)}{x^2} \cdot \frac{x + 6}{8}$$

$$\frac{\cancel{4}x(x + 2)(x + 6)}{2x^2} = \frac{(x + 2)(x + 6)}{2x}$$

d)  $\frac{x + 2}{x^2 - 1} \cdot \frac{x^2 + 2x - 3}{4x} \div \frac{x^2 + 5x + 6}{8x^2}$

$$\frac{x + 2}{(x + 1)(x - 1)} \cdot \frac{(x + 3)(x - 1)}{4x} \cdot \frac{8x^2}{(x + 3)(x + 2)}$$

$$\frac{2x}{x + 1}$$

17. Find the LCD

$$\frac{1}{x + 4}, \frac{3x}{2(x - 3)}, \frac{2x - 5}{x^2 + x - 12}$$

$$(x + 4)(x - 3)$$

$$\text{LCD} \rightarrow 2(x + 4)(x - 3)$$

18. Perform the indicated operation. Write your answer in simplest form.

a)  $\frac{x + 2}{x - 3} - \frac{3x - 5}{2x + 1} + \frac{3x^2 + 2x + 23}{2x^2 - 5x - 3}$

$$\frac{x + 2}{x - 3} - \frac{3x - 5}{2x + 1} + \frac{3x^2 + 2x + 23}{(2x + 1)(x - 3)}$$

$$\frac{(2x + 1)(x + 2) - (x - 3)(3x - 5) + 3x^2 + 2x + 23}{(2x + 1)(x - 3)}$$

$$\frac{2x^2 + 5x + 2 - (3x^2 - 14x + 15) + 3x^2 + 2x + 23}{(2x + 1)(x - 3)}$$

$$\frac{2x^2 + 21x + 10}{(2x + 1)(x - 3)}$$

b)  $\frac{(x + 2)(2x + 1)}{(x - 1)(2x + 1)} + \frac{(x + 3)(x - 1)}{(2x + 1)(x - 1)} - \frac{x^2 + 4x + 4}{(2x + 1)(x - 1)}$

$$\frac{2x^2 + 5x + 2 + x^2 + 2x - 3 - x^2 - 4x - 4}{(x - 1)(2x + 1)}$$

$$\frac{2x^2 + 3x - 5}{(x - 1)(2x + 1)}$$

$$\frac{(2x + 5)(x - 1)}{(x - 1)(2x + 1)}$$

$$\frac{2x + 5}{2x + 1}$$



19. Solve. Be sure to state any extraneous solutions.

$$\left[ \frac{6x^2}{x^2-16} - \frac{3x}{x+4} = \frac{4}{x-4} \right] (x-4)(x+4)$$

$$(x-4)(x+4)$$

$$6x^2 - 3x(x-4) = 4(x+4)$$

$$6x^2 - 3x^2 + 12x = 4x + 16$$

$$3x^2 + 8x - 16 = 0$$

$$(3x - 4)(x + 4) = 0$$

$$x = 4/3, \quad \cancel{-4}$$

$$\boxed{x = 4/3}$$

20. For the following questions, let  $f(x) = 5x^4 - 3x^2$  and  $g(x) = 4x^4$ . Perform the indicated operations and state the domain.

a)  $f(x) + g(x)$

$$= 5x^4 - 3x^2 + 4x^4$$

$$\boxed{= 9x^4 - 3x^2}$$

$$(-\infty, \infty)$$

b)  $f(x) - g(x)$

$$= 5x^4 - 3x^2 - 4x^4$$

$$\boxed{= x^4 - 3x^2}$$

$$(-\infty, \infty)$$

c)  $f(x) \cdot g(x)$

$$= (5x^4 - 3x^2) 4x^4$$

$$\boxed{= 20x^8 - 12x^6}$$

$$(-\infty, \infty)$$

d)  $\frac{f(x)}{g(x)}$

$$= \frac{5x^4 - 3x^2}{4x^4}$$

$$\boxed{= \frac{5x^2 - 3}{4x^2}}$$

$$(-\infty, 0) \cup (0, \infty)$$

e)  $f(g(x))$

$$= 5(4x^4)^4 - 3(4x^4)^2$$

$$= 5(256x^{16}) - 3(16x^8)$$

$$\boxed{= 1280x^{16} - 48x^8}$$

$$(-\infty, \infty)$$

21. For the following questions, let  $f(x) = \sqrt{x-3}$  and  $g(x) = 2x^2$ . Perform the indicated operations.

a) domain of  $f(x)$   $[3, \infty)$

b) domain of  $g(x)$   
 $(-\infty, \infty)$

c)  $f(g(4))$   
 $\sqrt{2(4)^2 - 3} = \sqrt{29}$

d)  $g(f(4))$   
 $2(\sqrt{4-3})^2$   
 $= 2\sqrt{1}^2 = \boxed{2}$

e)  $f(g(x))$   
 $= \sqrt{2x^2 - 3}$

f)  $g(f(x))$   
 $= 2\sqrt{x-3}^2$   
 $= 2(x-3)$   
 $= 2x - 6$

22. Solve the exponential equations:

but calc work for credit

a)  $5e^{3x} - 9 = 28$   
 $5e^{3x} = 37$   
 $e^{3x} = \frac{37}{5}$   
 $3x = \ln \frac{37}{5}$   
 $x = \frac{1}{3} \ln \frac{37}{5}$   
 $x \approx 0.667$

b)  $3^{x-2} + 4 = 11$   
 $3^{x-2} = 7$   
 $x-2 = \log_3 7$   
 $x = \log_3 7 + 2 \approx \boxed{3.771}$

23. Expand the expressions. Simplify when appropriate.

a)  $\log_3 \left( \frac{x^3}{81y^2} \right)$   
 $\log_3 x^3 - \log_3 81y^2$   
 $3\log_3 x - (\log_3 81 + \log_3 y^2)$

b)  $\log(10^{-5}z^8\sqrt{x^5})$   
 $-5\log 10 + 8\log z + \frac{5}{2}\log x$   
 $-5 + 8\log z + \frac{5}{2}\log x$

c)  $\log_{\frac{1}{2}} \sqrt{xy}$   
 $3\log_3 x - 4 - 2\log_3 y$   
 $\log_{\frac{1}{2}} x^{1/2} + \log_{\frac{1}{2}} y^{1/2}$

d)  $\ln xy$   
 $\ln x + \ln y$

$\frac{1}{2} \log_{\frac{1}{2}} x + \frac{1}{2} \log_{\frac{1}{2}} y$

24. Use the properties of logarithms to condense the expressions (write as one logarithm).

$$\begin{aligned} \text{a) } & 4 \log 2 + \log \frac{1}{2} - 3 \log c \\ & = \log 2^4 + \log \frac{1}{2} - \log c^3 \\ & = \log \frac{16(\frac{1}{2})}{c^3} = \boxed{\log \frac{8}{c^3}} \end{aligned}$$

$$\begin{aligned} \text{c) } & \ln 4xy^2 - 2 \ln x^2y \\ & = \ln \frac{4xy^2}{(x^2y)^2} = \ln \frac{4xy^2}{x^4y^2} = \boxed{\ln \frac{4}{x^3}} \end{aligned}$$

$$\begin{aligned} \text{b) } & 3 \ln a - 2 \ln b - \ln a \\ & = \ln a^3 - \ln b^2 - \ln a \\ & = \ln \frac{a^3}{b^2} - \ln a = \ln \frac{a^3}{b^2 a} \\ & = \ln \frac{a^2}{b^2} \end{aligned}$$

$$\begin{aligned} \text{d) } & \log_5 \sqrt[3]{x^2y} + \log_5 \sqrt[3]{xy^3} \\ & = \log_5 \sqrt[3]{x^3y^6} \\ & = \boxed{\log_5 xy^2} \end{aligned}$$

25. Solve the logarithmic equations. Check your answer for extraneous solutions.

$$\begin{aligned} \text{a) } & 5 = \log_x 32 \\ & x^5 = 32 \\ & x = \sqrt[5]{32} \\ & \boxed{x = 5} \end{aligned}$$

$$\begin{aligned} \text{b) } & \log_2 x + \log_2 (x+2) = 3 \\ & \log_2 (x^2 + 2x) = 3 \\ & 2^3 = x^2 + 2x \quad x = -1, 2 \\ & 0 = x^2 + 2x - 8 \quad \boxed{x = 2} \\ & 0 = (x+4)(x-2) \end{aligned}$$

$$\begin{aligned} \text{c) } & \log_4 x - \log_4 (x-3) = 1 \\ & \log_4 \frac{x}{x-3} = 1 \\ & 4^1 = \frac{x}{x-3} \quad \left. \begin{array}{l} 4x - 12 = x \\ 3x = 12 \end{array} \right\} \boxed{x = 4} \end{aligned}$$

$$\begin{aligned} \text{d) } & 2 \ln 3x = 4 \\ & \ln 3x = 2 \\ & e^2 = 3x \\ & \frac{1}{3} e^2 = x \\ & \boxed{x \approx 2.463} \end{aligned}$$

$$\begin{aligned} \text{e) } & \ln(x-2) + \ln(2x-3) = 2 \ln x \\ & \ln(x-2)(2x-3) = \ln x^2 \\ & 2x^2 - 4x - 3x + 6 = x^2 \\ & x^2 - 7x + 6 = 0 \\ & (x-6)(x-1) = 0 \\ & x = 6, 1 \\ & \boxed{x = 6} \end{aligned}$$

$$\begin{aligned} \text{f) } & \log(z-3) = 2 \\ & 10^2 = z - 3 \\ & \boxed{103 = z} \end{aligned}$$

★ Know equations for compound interest

$$A = Pe^{rt} \quad \text{and} \quad A = P \left(1 + \frac{r}{n}\right)^{nt}$$

