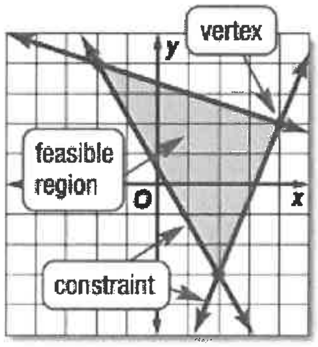
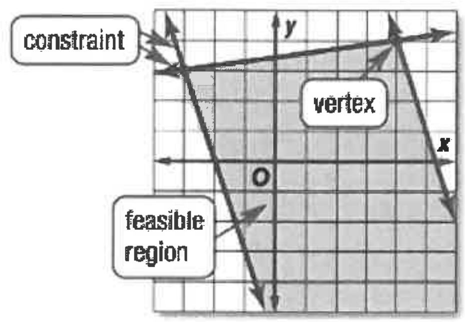
Linear Programming: a method for finding maximum or minimum values of a function over a given system of inequalities with each inequality representing a constraint.

Feasible Region: the solution set for a system of inequalities
* intersection of shading *

KeyConcept Feasible Regions



The feasible region is enclosed, or **bounded**, by the constraints. The maximum or minimum value of the related function *always* occurs at a vertex of the feasible region.



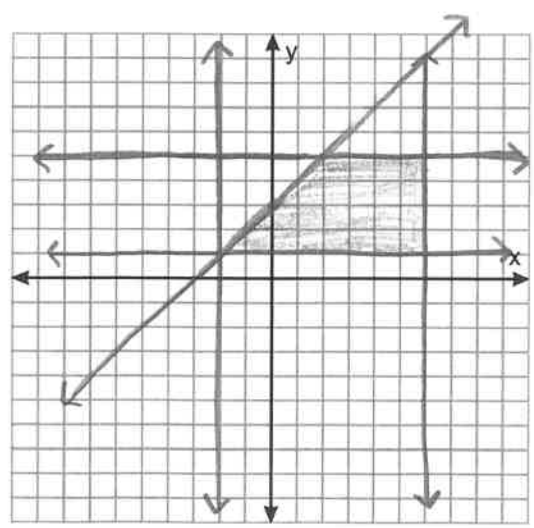
The feasible region is open and can go on forever. It is **unbounded**. Unbounded regions have either a maximum or a minimum.

1. Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values for this region.

$-2 \le x \le 6$
$1 \le y \le 5$
$y \le x + 3$
$f(x, y) = -5x + 2y$

vertices

(-2, 1) (6, 5)
(2, 5) (6, 1)



f(-2, 1) = 10 + 2 = 12 max
f(2, 5) = -10 + 10 = 0
f(6, 5) = -30 + 10 = -20
f(6, 1) = -30 + 2 = -28 min

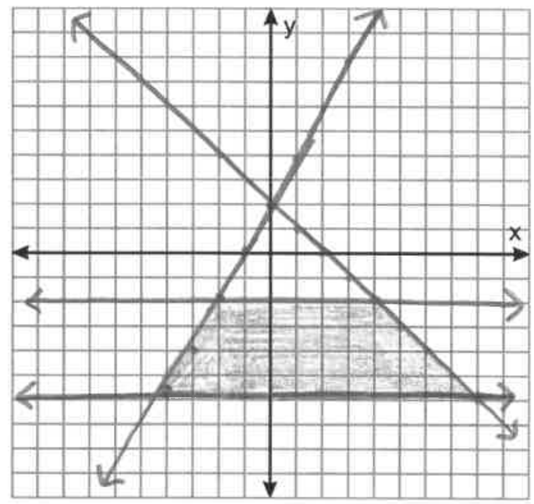
$$\boxed{\begin{array}{l}\text{Max value of } 12 \text{ @ } (-2,1) \\ \text{Min value of } -28 \text{ @ } (6,1)\end{array}}$$

2. Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values for this region.

$-6 \le y \le -2$
$y \le -x + 2$
$y \le 2x + 2$
$f(x, y) = 6x + 4y$

vertices

(-2, -2) (-4, -6)
(4, -2) (8, -6)



f(-2, -2) = -12 - 8 = -20
f(4, -2) = 24 - 8 = 16
f(-4, -6) = -24 - 24 = -48
f(8, -6) = 48 - 24 = 24

Max value of 24 @ (8, -6)

Min value of -48 @ (-4, -6)

3. Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values for this region.

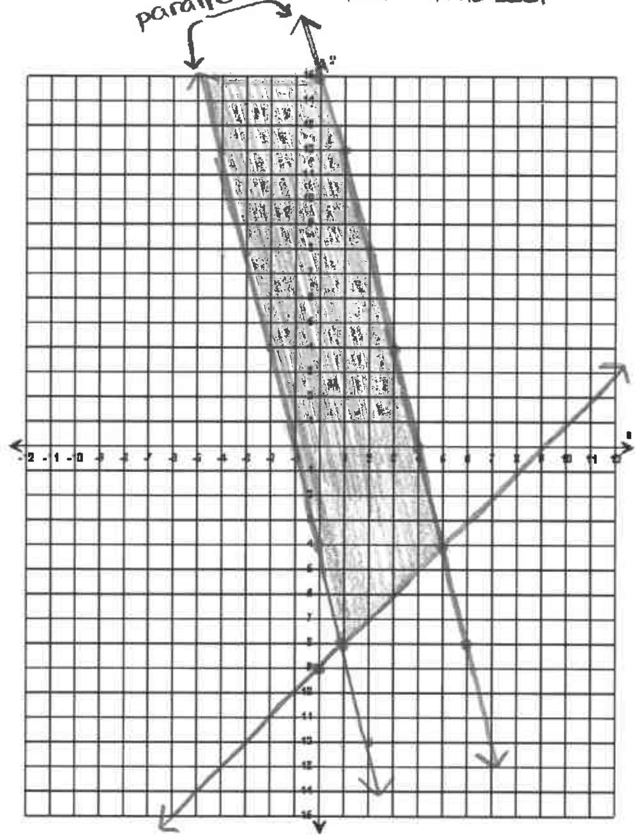
$y \ge x - 9$
$y \le -4x + 16$
$y \ge -4x - 4$
$f(x, y) = 10x + 7y$

unbounded so either a max or a min not both

vertices

(1, -8) (5, -4)

parallel will never intersect



f(1, -8) = 10 - 56 = -46

f(5, -4) = 50 - 28 = 22

* another value in feasible region would give greater value
ex: (-1, 16)
f(-1, 16) = -10 + 105 = 95

b/c unbounded no max value

Min value of -46 @ (1, -8)

4. An electronics company produces digital audio players and phones. A sign on the company bulletin board is shown.

Keeping Costs Down: We Can Do It!

Our Goal: Production per Shift			
Unit	Minimum	Maximum	Cost per Unit
audio	600	1500	\$55
phone	800	1700	\$95

The company is experiencing limits, or constraints, on production caused by customer demand, shipping, and the productivity of their factory. A system of inequalities can be used to represent these constraints. If at least 2000 items must be produced per shift, how many of each type should be made per shift to minimize costs?

a. Variables:

x = audio

y = phone

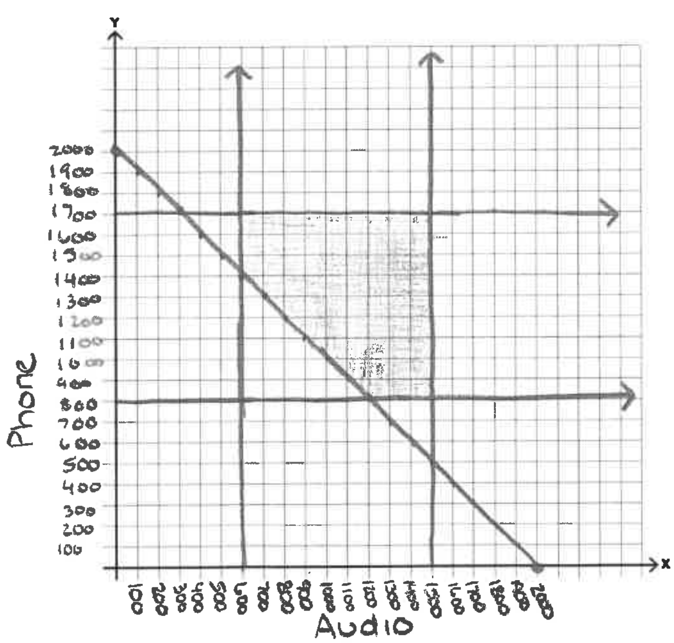
b. Inequalities:

$600 \leq x \leq 1500$

$800 \leq y \leq 1700$

$x + y \geq 2000$

c. Graph



d. Function to be minimized:

$f(x, y) = 55x + 95y$

vertices

(600, 1700) (1500, 1700)

(600, 1400) (1500, 800)

(1200, 800)

f(600, 1700) = 194,500

f(600, 1400) = 166,000

f(1500, 1700) = 244,000

f(1500, 800) = 158,500

f(1200, 800) = 142,000

e. Find minimum:

Produce 1200 audio players & 800 phones to minimize costs

5. Each week, Mackenzie can make 10 to 25 necklaces and 15 to 40 pairs of earrings. If she earns profits of \$3 on each pair of earrings and \$5 on each necklace, and she plans to sell at least 30 pieces of jewelry, how can she maximize profits?

x = necklaces

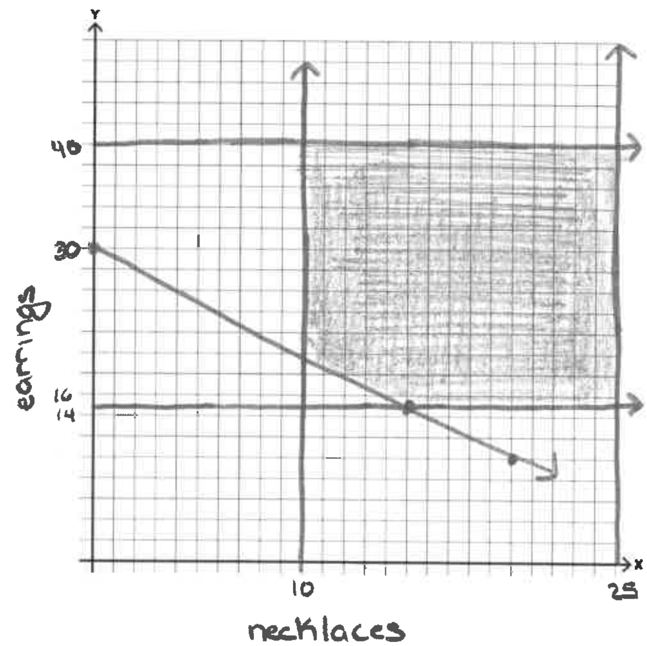
y = earrings

$10 \le x \le 25$

$15 \le y \le 40$

$x + y \ge 30$

$f(x, y) = 5x + 3y$



vertices

(15, 15)

(10, 20)

(10, 40)

(25, 40)

(25, 15)

$f(15, 15) = 120$

$f(10, 20) = 110$

$f(10, 40) = 170$

$f(25, 40) = 245 \rightarrow$ max

$f(25, 15) = 170$

max profits w/ 25 necklaces and 40 earrings