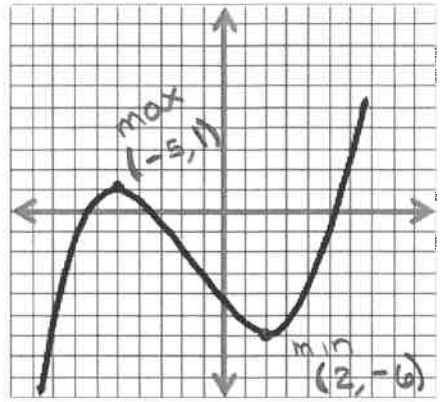
Relative (Local) Maximum: The turning point of the function that is higher than all nearby points

Relative (Local) Minimum: The turning point of the function that is lower than all nearby points

List all **relative extrema** (maxima/minima) as **ordered pairs**

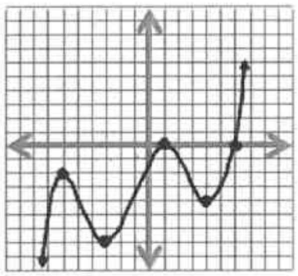
Real Zeros: Will also be the x-values of the x-intercepts



1.

Degree: Odd Even

Leading Coeff. sign: pos



Relative Maxima: (1, 0) & (-6, -2)

Relative Minima: (-3, 7) & (4, -4)

Real zeros: (1, 0) & (6, 0)

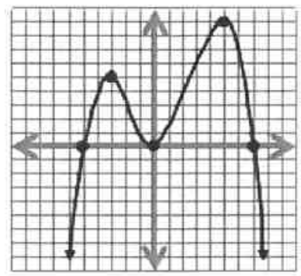
End Behavior: $x \to \infty$, $f(x) \to \infty$
$x \to -\infty$, $f(x) \to -\infty$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

2.

Degree: Odd Even

Leading Coeff. sign: neg



Relative Maxima: (-3, 5) & (5, 9)

Relative Minima: (0, 0)

Real zeros: (-5, 0) & (0, 0) & (7, 0)

End Behavior: $x \to \infty$, $f(x) \to -\infty$
$x \to -\infty$, $f(x) \to -\infty$

Domain: $(-\infty, \infty)$ Range: $(-\infty, 9]$

3. $f(x) = -x^3 + 2x^2 + 15x + 2$

Degree: 3 (odd)

Leading Coefficient Value: -1 (neg)

Total Number of Zeros: 3

End Behavior: $x \to \infty$ $f(x) \to -\infty$
$x \to -\infty$ $f(x) \to \infty$

Domain: $(-\infty, \infty)$

4. $f(x) = 2x^4 - 3x^3 - 2x^2 + 7x + 1$

Degree: 4 (even)

Leading Coefficient Value: 2 (pos)

Total Number of Zeros: 4

End Behavior: $x \to \infty$ $f(x) \to \infty$
$x \to -\infty$ $f(x) \to \infty$

Domain: $(-\infty, \infty)$

Honors Algebra 2
5.3 Polynomial Functions
5.4 Analyzing Graphs of Polynomial Functions

Polynomial Function: A monomial or a sum of monomials.

Written in the form: $f(x) = ax^n + bx^{n-1} + cx^{n-2} + \ldots + dx + e$

The exponents are whole numbers and the coefficients are real numbers.

Example: Polynomial or not? Explain:

$f(x) = 3x^2 + 2x - 10$	$f(x) = x^{\frac{1}{2}} - 4ix - 10$	$f(x) = -2x^{10} + \sqrt{\pi}\, x^7 + 4x - 1$
polynomial	not a polynomial coeff. not ℝ and exp. not whole #	polynomial

Degree: The largest exponent (degree) of the variable x.

* The degree indicates the number of zero's for the polynomial (real & imaginary)*

Leading Coefficient: The coefficient of the term with the largest exponent.

Example:

$f(x) = -x^4 + 3x^3 - x + 1$

Degree: 4 **Leading Coefficient:** −1 **Total Number of Zeros:** 4

End Behavior: the direction the graph goes as $x \to -\infty$ (x approaches negative infinity) and as $x \to +\infty$ (x approaches positive infinity) * depends on degree and leading coefficient

	Odd Degree	**Even Degree**
Positive Leading Coefficient	pos coeff **Example:** $2x^3$ $x \to +\infty,\ f(x) \to$ ∞ $x \to -\infty,\ f(x) \to$ $-\infty$	pos coeff **Example:** $2x^2$ $x \to +\infty,\ f(x) \to$ ∞ $x \to -\infty,\ f(x) \to$ ∞
Negative Leading Coefficient	neg coeff **Example:** $-2x^3$ $x \to +\infty,\ f(x) \to$ $-\infty$ $x \to -\infty,\ f(x) \to$ ∞	neg coeff **Example:** $-2x^2$ $x \to +\infty,\ f(x) \to$ $-\infty$ $x \to -\infty,\ f(x) \to$ $-\infty$
	If the degree is ODD, then the tails go in opposite directions.	**If the degree is EVEN, then the tails go in the same direction.**

EXAMPLES:
State End Behavior

3) $f(x) = \boxed{3x^4} + 2x^2 - 1$

$x \to +\infty,\ f(x) \to$ ∞

$x \to -\infty,\ f(x) \to$ ∞

4) $f(x) = \boxed{-x^5} + 3x^4 - 2x^3 - 4x - 1$

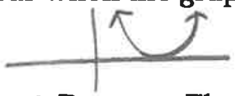
$x \to +\infty,\ f(x) \to$ $-\infty$

$x \to -\infty,\ f(x) \to$ ∞

Real Zeros: Real zeros exist when the graph ___crosses___ the x-axis.

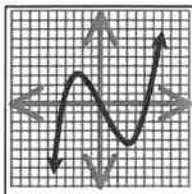
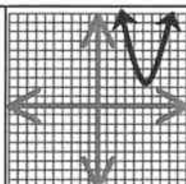
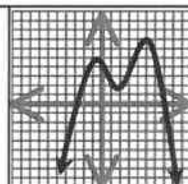
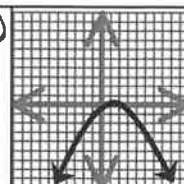
Imaginary Zeros: Imaginary zeros exist when the graph ___doesn't___ touch the x-axis   ex) $y = x^2 + 4$

Double Zeros: Occur when the graph ___touches___ the x-axis then turns away.
* bounces



Polynomial of Least Degree: The smallest degree of a polynomial that will fit the given graph or zeros.

Example: State the types of zeros contained in the following polynomials of least degree.

 3rd degree (pos coeff)	 2nd degree (pos coeff)	 (neg coeff) 4th degree	 2nd degree (neg coeff)
$\mathbb{R}$ zeros	Imaginary zeros	2 $\mathbb{R}$ zeros 2 imaginary	Double zero

A polynomial function is in **Standard Form** if its terms are written in descending order of ___degree___ from left to right.

Example: Decide whether the function is a polynomial function. If so, write it in **standard form** and state its **degree,** and **leading coefficient**.

1. $f(x) = x^4 - \frac{1}{4}x^7 + 2$

   $f(x) = -\frac{1}{4}x^7 + x^4 + 2$

   7th degree
   -1/4 leading coeff

2. $f(x) = x^3 - 0.3x^{-1}$

   not a polynomial
   ↳ degrees are not whole #'s

3. $f(x) = 7x - \sqrt{3} + \pi x^2$

   $f(x) = \pi x^2 + 7x - \sqrt{3}$

   2nd degree
   π leading coeff.

4. $f(x) = x + 2^x$

   not a polynomial
   ↳ degree not whole #

1) let $f(x) = 3x^3 + 4x^2 + 2$

Find $f(3c)$

$$f(3c) = 3(3c)^3 + 4(3c)^2 + 2$$
$$= 3(27c^3) + 4(9c^2) + 2$$
$$= 81c^3 + 36c^2 + 2$$

2) let $g(x) = x^2 - 5$

Find $g(w-5)$

$$g(w-5) = (w-5)^2 - 5$$
$$= w^2 - 10w + 25 - 5$$
$$= w^2 - 10w + 20$$

3) let $f(x) = 3x - 4$ and $g(x) = x^2 + 4x + 9$

Find $f(2y+1) - g(y^2)$

$$f(2y+1) - g(y^2) = 3(2y+1) - 4 - [(y^2)^2 + 4(y^2) + 9]$$
$$= 6y + 3 - 4 - (y^4 + 4y^2 + 9)$$
$$= 6y - 1 - y^4 - 4y^2 - 9$$
$$= -y^4 - 4y^2 + 6y - 10$$