

5.5 Solving Polynomial Equations
Honors Algebra 2

Difference of Squares	$a^2 - b^2$	$(a - b)(a + b)$
Sum of Cubes	$a^3 + b^3$	$(a + b)(a^2 - ab + b^2)$
Difference of Cubes	$a^3 - b^3$	$(a - b)(a^2 + ab + b^2)$

"plus minus, plus"

"minus, plus, plus"

Factor each:

<p>1. $27 - r^3 = (3)^3 - (r)^3$ $a = 3 \quad b = r$</p> <p>$(3 - r)(3^2 + 3r + r^2)$ $= (3 - r)(9 + 3r + r^2)$</p>	<p>2. $8m^3 - 27n^3 = (2m)^3 - (3n)^3$ $a = 2m \quad b = 3n$</p> <p>$= (2m - 3n)((2m)^2 + (2m)(3n) + (3n)^2)$ $= (2m - 3n)(4m^2 + 6mn + 9n^2)$</p>
<p>3. $x^3 + 64 = (x)^3 + (4)^3$ $a = x \quad b = 4$</p> <p>$= (x + 4)(x^2 - 4x + 4^2)$ $= (x + 4)(x^2 - 4x + 16)$</p>	<p>4. $16x^4 + 54xy^3$ * GCF</p> <p>$= 2x(8x^3 + 27y^3)$ $= 2x((2x)^3 + (3y)^3)$ $= 2x(2x + 3y)((2x)^2 - (2x)(3y) + (3y)^2)$ $= 2x(2x + 3y)(4x^2 - 6xy + 9y^2)$</p>

Factor by grouping:

<p>$4x^3 + 2x^2 - 2x - 1$</p> <p>$= (4x^3 + 2x^2) + (-2x - 1)$</p> <p>$= 2x^2(2x + 1) - 1(2x + 1)$</p> <p>* $2x + 1$ GCF</p> <p>$= (2x + 1)(2x^2 - 1)$</p>	<p>$10ab - 6b + 35a - 21$</p> <p>$= (10ab - 6b) + (35a - 21)$</p> <p>$= 2b(5a - 3) + 7(5a - 3)$</p> <p>$= (5a - 3)(2b + 7)$</p>
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$8r^3 - 64r^2 + r - 8$ $= (8r^3 - 64r^2) + (r - 8)$ $= 8r^2(r - 8) + (r - 8)$ $= (r - 8)(8r^2 + 1)$	$(4x^6 + 36)(x^6y - 9y)$ $= 4(x^6 + 9) + (-y)(x^6 + 9)$ $= (x^6 + 9)(4 - y)$
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Factor each expression completely:

<p>1. $3y^5 - 75y^3$</p> $= 3y^3(y^2 - 25)$ $= 3y^3(y - 5)(y + 5)$	<p>2. $(x^3 - 3x^2)(16x + 48)$</p> $= x^2(x - 3) + (-16)(x - 3)$ $= (x - 3)(x^2 - 16)$ $= (x - 3)(x - 4)(x + 4)$
<p>3. $2x^4 + 128x$</p> $= 2x(x^3 + 64)$ $= 2x(x + 4)(x^2 - 4x + 16)$ <p style="text-align: center;"> prime, so done factoring </p>	<p>4. $2p^8 + 10p^5 + 12p^2$</p> $= 2p^2(p^6 + 5p^3 + 6)$ $= 2p^2(p^3 + 3)(p^3 + 2)$

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$$0 = (x-a)(x-b)$$

$$0 = x-a \quad \text{or} \quad 0 = x-b$$

Use the zero product property to solve:

* Notice # of solutions and degree!

5. $3x^5 + 15x = 18x^3$

$$3x^5 - 18x^3 + 15x = 0$$

$$3x(x^4 - 6x^2 + 5) = 0$$

$$3x(x^2 - 5)(x^2 - 1) = 0$$

$$3x(x^2 - 5)(x-1)(x+1) = 0$$

$$3x = 0 \quad x^2 - 5 = 0 \quad x+1 = 0 \quad x-1 = 0$$

$$x = 0 \quad x = \pm\sqrt{5} \quad x = -1 \quad x = 1$$

$$x = 0, \pm\sqrt{5}, \pm 1$$

6. $5b^3 + 15b^2 + 12b = -36$

$$(5b^3 + 15b^2) + (12b + 36) = 0$$

$$5b^2(b+3) + 12(b+3) = 0$$

$$(5b^2 + 12)(b+3) = 0$$

$$5b^2 + 12 = 0 \quad b+3 = 0$$

$$b = \pm\sqrt{-12/5} \quad b = -3$$

$$b = \pm i\sqrt{\frac{12}{5}}$$

$$b = \pm i\sqrt{\frac{60}{5}}$$

$$b = -3, \pm i\sqrt{60/5}$$

7. $(x^6 - 4x^4)(9x^2 + 36) = 0$

$$x^4(x^2 - 4) - 9(x^2 - 4) = 0$$

$$(x^4 - 9)(x^2 - 4) = 0$$

$$(x^2 - 3)(x^2 + 3)(x-2)(x+2) = 0$$

$$x^2 - 3 = 0 \quad x^2 + 3 = 0 \quad x - 2 = 0 \quad x + 2 = 0$$

$$x = \pm\sqrt{3} \quad x = \pm i\sqrt{3} \quad x = 2 \quad x = -2$$

$$x = \pm\sqrt{3}, \pm i\sqrt{3}, \pm 2$$

8. $x^3 + 1000 = 0$

$$(x+10)(x^2 - 10x + 100) = 0$$

*prime

$$x+10 = 0$$

$$x = -10$$

$$x = \frac{10 \pm \sqrt{10^2 - 4(1)(100)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 - 400}}{2}$$

$$= \frac{10 \pm \sqrt{-300}}{2}$$

$$= \frac{10 \pm 10i\sqrt{3}}{2}$$

$$= 5 \pm 5i\sqrt{3}$$

$$x = -10, 5 \pm 5i\sqrt{3}$$

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Factor the following:

<p>9. $(8ax + 4bx + 4cx) + (6ay + 3by + 3cy)$ $= 4x(2a + b + c) + 3y(2a + b + c)$ $= (4x + 3y)(2a + b + c)$</p>	<p>10. $20fy - 16fz + 15gy + 8hz - 10hy - 12gz$ $= (20fy + 15gy - 10hy) + (-16fz + 8hz - 12gz)$ $= 5y(4f + 3g - 2h) +$ $(-4)(4f - 2h + 3g)$ $= 5y(4f + 3g - 2h) - 4(4f + 3g - 2h)$ $= (5y - 4)(4f + 3g - 2h)$</p>
<p>11. $x^6 - y^6$ $= (x^2)^3 - (y^2)^3$ $= (x^2 - y^2)(x^4 + x^2y^2 + y^4)$ $= (x^2 - y^2)(x^4 + x^2y^2 + y^4)$ $= (x - y)(x + y)(x^4 + x^2y^2 + y^4)$ prime 5</p>	<p>12. $(a^3x^2 - 6a^3x + 9a^3) + (b^3x^2 + 6b^3x - 9b^3)$ $= a^3(x^2 - 6x + 9) + (-b^3)(x^2 - 6x + 9)$ $= (a^3 - b^3)(x^2 - 6x + 9)$ $= (a - b)(a^2 + ab + b^2)(x - 3)(x - 3)$ prime 5</p>

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13. $150n^8 + 40n^4 - 15$

$5(30n^8 + 8n^4 - 3)$

cannot be factored
further

14. $y^8 + 12y^3 + 8$

cannot be factored

