1. Solve each equation. State the number and type of roots.

   a. $0 = x^2 + 6x + 9$

   $0 = (x+3)(x+3)$

   $x = -3$

   double zero

   b. $x^3 + 25x = 0$

   $x(x^2+25) = 0$

   $x = 0, \pm 5i$

   one real and 2 imaginary

A nth degree polynomial has ___n___ zeros.

KeyConcept Descartes' Rule of Signs

Let $P(x) = a_n x^n + \cdots + a_1 x + a_0$ be a polynomial function with real coefficients. Then

- the number of positive real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and
- the number of negative real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this by an even number.

2. State the possible number of positive, real zeros, negative real zeros, and imaginary zeros of $f(x) = x^6 + 3x^5 - 4x^4 - 6x^3 + x^2 - 8x + 5$. 6 zeros

positive

$f(x) = x^6 + 3x^5 - 4x^4 - 6x^3 + x^2 - 8x + 5$

1 2 3 4

4 changes in sign

negative

$f(-x) = (-x)^6 + 3(-x)^5 - 4(-x)^4 - 6(-x)^3 + (-x)^2 - 8(-x) + 5$

$f(-x) = x^6 - 3x^5 - 4x^4 + 6x^3 + x^2 + 8x + 5$

1 2

2 changes in sign

4 pos	2 neg	
2 pos	2 neg	2 imaginary
0 pos	2 neg	4 imaginary
0 pos	0 neg	6 imaginary
4 pos	0 neg	2 imaginary
2 pos	0 neg	4 imaginary

3. State the possible number of positive, real zeros, negative real zeros, and imaginary zeros of $f(x) = 2x^5 + x^4 + 3x^3 - 4x^2 - x + 9$. 5 zeros

pos

$f(x) = 2x^5 + x^4 + 3x^3 - 4x^2 - x + 9$

2 changes in sign

neg

$f(-x) = 2(-x)^5 + (-x)^4 + 3(-x)^3 - 4(-x)^2 - (-x) + 9$

$= -2x^5 + x^4 - 3x^3 - 4x^2 + x + 9$

3 changes in sign

2 pos 3 neg
0 pos 3 neg 2 imaginary
2 pos 1 neg 2 imaginary
0 pos 1 neg 4 imaginary

4. State the possible number of positive, real zeros, negative real zeros, and imaginary zeros of $f(x) = -x^5 + 14x^3 + 18x - 36$. 5 zeros

pos

$f(x) = -x^5 + 14x^3 + 18x - 36$

2 changes

neg

$f(-x) = -(-x)^5 + 14(-x)^3 + 18(-x) - 36$

$= x^5 - 14x^3 - 18x - 36$

1 change

2 pos 1 neg 2 imaginary
0 pos 1 neg 4 imaginary

5. State the possible number of positive, real zeros, negative real zeros, and imaginary zeros of $f(x) = x^4 - 2x^2 - 5x + 19$. 4 zeros

pos

$f(x) = x^4 - 2x^2 - 5x + 19$

2 changes

neg

$f(-x) = (-x)^4 - 2(-x)^2 - 5(-x) + 19$

$= x^4 - 2x^2 + 5x + 19$

2 changes

2 pos 2 neg
0 pos 2 neg 2 imaginary
2 pos 0 neg 2 imaginary
0 pos 0 neg 4 imaginary

KeyConcept Complex Conjugates Theorem

Words	Let a and b be real numbers, and $b \neq 0$. If $a + bi$ is a zero of a polynomial function with real coefficients, then $a - bi$ is also a zero of the function.
Example	If $3 + 4i$ is a zero of $f(x) = x^3 - 4x^2 + 13x + 50$, then $3 - 4i$ is also a zero of the function.

6. Write a polynomial function of least degree with integral coefficients, the zeros of which include -1 and $5 - i$.

so $5+i$ also zero

$f(x) = (x+1)(x-(5-i))(x-(5+i))$

$= (x+1)(x-5+i)(x-5-i)$

$= (x+1)((x-5)+i)((x-5)-i)$ ★ difference of 2 perfect squares

$= (x+1)((x-5)^2 - i^2)$ $\quad (x-3)(x+3)$

$= (x+1)((x^2-10x+25)-(-1))$ $\quad = x^2-9$

$= (x+1)(x^2-10x+26) = x^3 - 10x^2 + 26x + x^2 - 10x + 26 = \boxed{x^3 - 9x^2 + 16x + 26}$

7. Write a polynomial function of least degree with integral coefficients, the zeros of which include -1 and $1 + 2i$. $1-2i$

$f(x) = (x+1)(x-(1+2i))(x-(1-2i))$

$= (x+1)(x-1-2i)(x-1+2i)$

$= (x+1)[(x-1)-2i][(x-1)+2i]$

$= (x+1)[(x-1)^2 - (2i)^2]$

$= (x+1)(x^2-2x+1-4i^2)$

$= (x+1)(x^2-2x+1+4)$

$= (x+1)(x^2-2x+5)$

$= x^3 - 2x^2 + 5x + x^2 - 2x + 5$

$\boxed{= x^3 - x^2 + 3x + 5}$

8. Write a polynomial function of least degree with integral coefficients, the zeros of which include -3, 1, and $-3i$ and $3i$

$f(x) = (x+3)(x-1)(x-(-3i))(x-3i)$

$= (x+3)(x-1)(x+3i)(x-3i)$

$= (x+3)(x-1)(x^2-9i^2)$

$= (x+3)(x-1)(x^2+9)$

$= (x^2+2x-3)(x^2+9)$

$= x^4+9x^2+2x^3+18x-3x^2-27$

$= \boxed{x^4+2x^3+6x^2+18x-27}$