1. Given $f(x) = x^2 - 4$ and $g(x) = 2x + 1$, find each function. Indicate any restrictions in the domain.

a. $(f + g)(x)$

$(f+g)(x) = (x^2-4) + (2x+1)$

$= \boxed{x^2 + 2x - 3}$

domain $(-\infty, \infty)$

c. $(f \cdot g)(x)$

$(f \cdot g)(x) = (x^2-4)(2x+1)$

$= 2x^3 - 8x + x^2 - 4$

$= \boxed{2x^3 + x^2 - 8x - 4}$

domain $(-\infty, \infty)$

b. $(f - g)(x)$

$(f-g)(x) = (x^2-4) - (2x+1)$

$= x^2 - 4 - 2x - 1$

$= \boxed{x^2 - 2x - 5}$

domain $(-\infty, \infty)$

d. $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \boxed{\dfrac{x^2-4}{2x+1}}$

domain: $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

2. Given $f(x) = x^2 + 7x + 12$ and $g(x) = 3x - 4$, find each function. Indicate any restrictions in the domain.

a. $(f + g)(x)$

$(f+g)(x) = (x^2 + 7x + 12) + (3x-4)$

$= \boxed{x^2 + 10x + 8}$

domain $(-\infty, \infty)$

b. $(f - g)(x)$

$(f-g)(x) = (x^2+7x+12) - (3x-4)$

$= x^2 + 7x + 12 - 3x + 4$

$= \boxed{x^2 + 4x + 16}$

doman $(-\infty, \infty)$

c. $(f \cdot g)(x)$

$(f \cdot g)(x) = (x^2+7x+12)(3x-4)$

$= 3x^3 + 21x^2 + 36x - 4x^2$
$-28x - 48$

$= \boxed{3x^3 + 17x^2 + 8x - 48}$

domain $(-\infty, \infty)$

d. $\left(\frac{f}{g}\right)(x)$

$(f/g)(x) = \boxed{\dfrac{x^2+7x+12}{3x-4}}$

domain: $(-\infty, 4/3) \cup (4/3, \infty)$

Composition of Functions:

Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composition function $f \circ g$ can be described by: $[f \circ g](x) = f[g(x)]$

3. For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

a. $f = \{(1,8), (0,13), (15,11), (14,9)\}$
$g = \{(8,15), (5,1), (10,14), (9,0)\}$

$(f \circ g)(x)$ * range of g subset of domain of f ✓

$f(g(8)) = f(15) = 11$
$f(g(5)) = f(1) = 8$
$f(g(10)) = f(14) = 9$
$f(g(9)) = f(0) = 13$

$(f \circ g)(x) = \{(8,11), (5,8), (10,9), (9,13)\}$
domain: $\{8, 5, 10, 9\}$
range: $\{11, 8, 9, 13\}$

$(g \circ f)(x)$

$g(f(1)) = g(8) = 15$
$g(f(0)) = g(13)$
$g(f(15)) = g(11)$
$g(f(14)) = g(9) = 0$

$(g \circ f)(x) = \{(1, 15), (14, 0$
domain: $\{8, 5, 1, 9, 9$
range: $\{11, 8, 9, 13\}$

b. $f = \{(3,-2), (-1,-5), (4,7), (10,8)\}$
$g = \{(4,3), (2,-1), (9,4), (3,10)\}$

$(f \circ g)(x)$

$f(g(4)) = f(3) = -2$
$f(g(2)) = f(-1) = -5$
$f(g(9)) = f(4) = 7$
$f(g(3)) = f(10) = 8$

$(f \circ g)(x) = \{(4,-2), (2,-5), (9,7), (3,8)\}$
domain: $\{4, 2, 9, 3\}$
range: $\{-2, -5, 7, 8\}$

$(g \circ f)(x)$

$g(f(3)) = g(-2)$
$g(f(-1)) = g(-5)$
$g(f(4)) = g(7)$
$g(f(10)) = g(8)$

$(g \circ f)(x)$ is undefined for $x = 3$, $x = 4$, $x = -1$, and $x = 10$

c. $f(x) = 2x - 5$ and $g(x) = 4x$

$(f \circ g)(x) = f(g(x))$
$= 2(4x) - 5$
$= \boxed{8x - 5}$

$(g \circ f)(x) = g(f(x))$
$= 4(2x - 5)$
$= \boxed{8x - 20}$

d. $f(x) = x^2 + 2$ and $g(x) = x - 6$

$(f \circ g)(x) = (x-6)^2 + 2$
$= x^2 - 12x + 36 + 2$
$= \boxed{x^2 - 12x + 38}$

$(g \circ f)(x) = g(f(x))$
$= (x^2 + 2) - 6$
$= \boxed{x^2 - 4}$

e. $f(x) = -3x$ and $g(x) = -x + 8$

$(f \circ g)(x) = -3(-x + 8)$
$= \boxed{3x - 24}$

$(g \circ f)(x) = -(-3x) + 8$
$= \boxed{3x + 8}$

f. $f(x) = x - 4$ and $g(x) = x^2 - 10$

$(f \circ g)(x) = f(g(x))$
$= (x^2 - 10) - 4$
$= \boxed{x^2 - 14}$

$(g \circ f)(x) = (x-4)^2 - 10$
$= x^2 - 8x + 16 - 10$
$= \boxed{x^2 - 8x + 6}$

g. $f(x) = x^2 + 2x$ and $g(x) = 4x$

$(f \circ g)(x) = (4x)^2 + 2(4x)$
$= \boxed{16x^2 + 8x}$

$(g \circ f)(x) = 4(x^2 + 2x)$
$= \boxed{4x^2 + 8x}$

4. If $f(x) = 5x$ and $g(x) =- 2x + 1$, and $h(x) = x^2 + 6x + 8$, find each value:

a. $g[h(3)]$

$= g((3)^2 + 6(3) + 8)$

$= g(9 + 18 + 8)$

$= g(35)$

$= -2(35) + 1$

$= -70 + 1$

$= \boxed{-69}$

b. $h[g(2)]$

$= h(-2(2) + 1)$

$= h(-4 + 1)$

$= h(-3)$

$= (-3)^2 + 6(-3) + 8$

$= 9 - 18 + 8$

$= \boxed{-1}$

c. $f[h(a + 4)]$

$= f[(a+4)^2 + 6(a+4) + 8]$

$= f(a^2 + 8a + 16 + 6a + 24 + 8)$

$= f(a^2 + 14a + 48)$

$= 5(a^2 + 14a + 48)$

$= \boxed{5a^2 + 70a + 240}$