Inverse Relation: the set of ordered pairs obtained by exchanging the coordinates

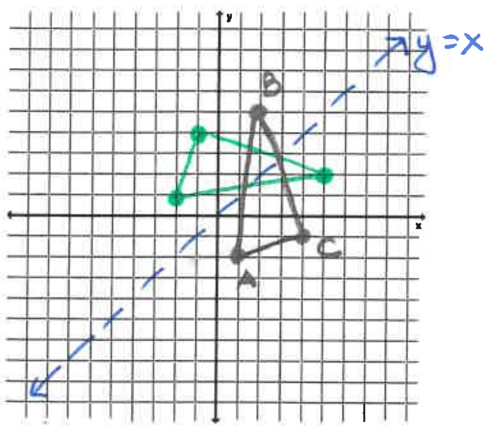
The domain of a relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse.

KeyConcept Inverse Relations

Words	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Example	A and B are inverse relations.
	$A = \{(1, 5), (2, 6), (3, 7)\}$ $\quad B = \{(5, 1), (6, 2), (7, 3)\}$

*swap x and y

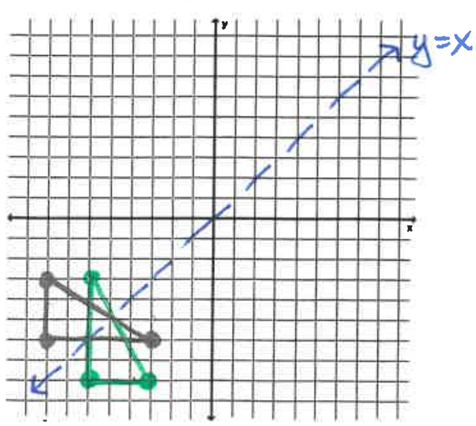
1. The vertices of ΔABC can be represented by the relation $\{(1, -2), (2, 5), (4, -1)\}$. Find the inverse of this relation. Describe the graph of the inverse.



inverse: $\{(-2, 1), (5, 2), (-1, 4)\}$

*reflected over $y=x$

2. The ordered pairs of the relation $\{(-8, -3), (-8, -6), (-3, -6)\}$ are the coordinates of the vertices of a right triangle. Find the inverse of this relation. Describe the graph of the inverse.



inverse: $\{(-3, -8), (-6, -8), (-6, -3)\}$

*reflected over $y=x$

6.2 Inverse Functions and Relations
Honors Algebra 2

Notation for an inverse: $f^{-1}(x)$ * doesn't mean exponent -1 *

When the inverse of a function is a function, the original function is one - to - one.

The horizontal line test can be used to determine whether the inverse of a function is also a function.

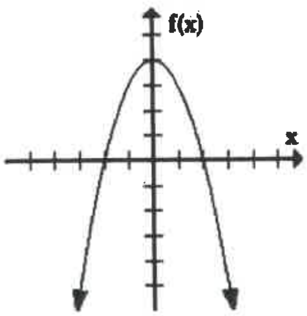
* use vertical line test to determine graph is a function

3. Determine whether the inverse of the functions below will also be functions.

* restrict domain w/ $y = x^2$, $x \geq 0$ to take inverse

$f^{-1}(x) = \sqrt{x}$

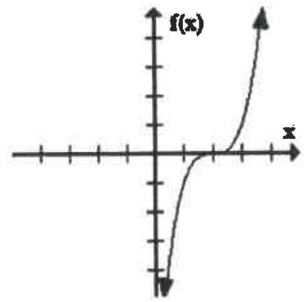
a.



Not one - to - one

inverse not a function

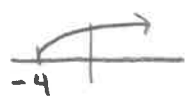
b.



one - to - one

inverse is a function

c. $f(x) = \sqrt{x+4}$



is one - to - one

inverse is a function

d. $f(x) = x^2 - 2$

Not one - to - one

inverse not a function

The inverse of a function can be found by swapping the domain and range

* swap x and y

solve for y

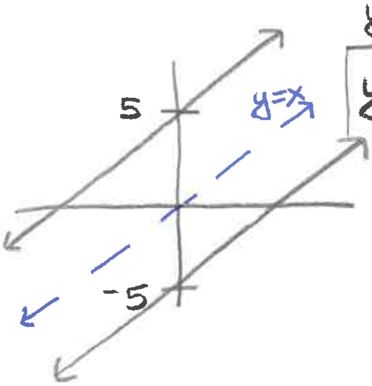
4. Find the inverse of each function. Then graph the function and its inverse.

a. $y = x + 5$

$x = y + 5$

$y = x - 5$

$\boxed{y^{-1} = x - 5}$



b. $f(x) = x^2 + 1$

* not one-to-one

restrict domain

$x \geq 0$

$x = y^2 + 1$

$x - 1 = y^2$

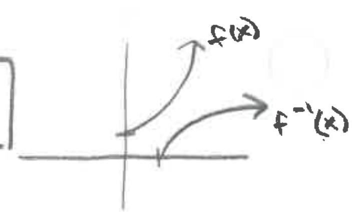
$\pm\sqrt{x-1} = y$

$\sqrt{x-1} = y$

$\boxed{f^{-1}(x) = \sqrt{x-1}}$

if $x \geq 0$ for $f(x)$

then $y \geq 0$ for $f^{-1}(x)$



c. $y = \frac{x-3}{4}$

$x = \frac{y-3}{4}$

$4x = y - 3$

$4x + 3 = y$

$\boxed{y^{-1} = 4x - 3}$

d. $y = 3x^2$

$* \; x \geq 0$

$x = 3y^2$

$\frac{1}{3}x = y^2$

$\pm\sqrt{\frac{1}{3}x} = y$

$\sqrt{1/3x} = y$

$\boxed{y^{-1} = \sqrt{1/3x}}$

You can determine if functions are inverses by finding both of their compositions.

If both compositions equal x, then they are inverses.

$f(g(x)) = x$
$g(f(x)) = x$

KeyConcept Inverse Functions

Words	Two functions f and g are inverse functions if and only if both of their compositions are the identity function.
Symbols	$f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

5. Verify that the two functions are inverses:
   a. $f(x) = 3x + 9$ and $g(x) = \frac{1}{3}x - 3$

$f(g(x)) = 3(1/3x - 3) + 9$

$= x - 9 + 9$

$= x$ ✓

$g(f(x)) = \frac{1}{3}(3x + 9) - 3$

$= x + 3 - 3$

$= x$ ✓

f(x) and g(x) are inverses

b. $f(x) = 4x^2$ and $g(x) = 2\sqrt{x}$

$f(g(x)) = 4(2\sqrt{x})^2$

$= 4(4x)$

$= 16x$

not inverses

c. $f(x) = 3x - 3$ and $g(x) = \frac{1}{3}x + 4$

$f(g(x)) = 3(\frac{1}{3}x + 4) - 3$

$= x + 12 - 3$

$= x + 9$

not inverses

d. $f(x) = 2x^2 - 1$ and $g(x) = \sqrt{\frac{x+1}{2}}$

$f(g(x)) = 2\left(\sqrt{\frac{x+1}{2}}\right)^2 - 1$

$= 2\left(\frac{x+1}{2}\right) - 1$

$= x + 1 - 1$

$= x$ ✓

$g(f(x)) = \sqrt{\frac{(2x^2-1)+1}{2}}$

$= \sqrt{\frac{2x^2}{2}}$

$= \sqrt{x^2}$

$= x$ ✓

inverses