

Finding the square root of a number and squaring a number are inverse operations. To find the square root of a number a , you must find a number with a square of a . Similarly, the inverse of raising a number to the n th power is finding the n^{th} root of a number.

Powers	Factors	Words	Roots
$x^3 = 64$	$4 \cdot 4 \cdot 4 = 64$	4 is a cube root of 64.	$\sqrt[3]{64} = 4$
$x^4 = 625$	$5 \cdot 5 \cdot 5 \cdot 5 = 625$	5 is a fourth root of 625.	$\sqrt[4]{625} = 5$
$x^5 = 32$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	2 is a fifth root of 32.	$\sqrt[5]{32} = 2$
$a^n = b$	$\underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_n = b$ n factors of a	a is an n th root of b .	$\sqrt[n]{b} = a$

KeyConcept Definition of n th Root

Words For any real numbers a and b , and any positive integer n , if $a^n = b$, then a is an n th root of b .

Example Because $(-3)^4 = 81$, -3 is a fourth root of 81 and 3 is a principal root.

Symbol that indicates n th root:



Some numbers have more than one real n th root.

ex) 64 has two square roots $8^2 = 64$ and $(-8)^2 = 64$

When there is more than one real root and n is even, the nonnegative root is called the principal root.

8 principal root for $\sqrt{64}$

Examples of n th roots:

- $\sqrt{25} = 5$ $\sqrt{25}$ indicates the principal square root of 25.
- $-\sqrt{25} = -5$ $-\sqrt{25}$ indicates the opposite of the principal square root of 25.
- $\pm\sqrt{25} = \pm 5$ $\pm\sqrt{25}$ indicates both square roots of 25.

1. Simplify:

$$\begin{aligned} \text{a. } & \pm \sqrt{16y^4} \\ & = \pm \sqrt{4^2 (y^2)^2} \\ & = \boxed{\pm 4y^2} \end{aligned}$$

$$\begin{aligned} \text{b. } & -\sqrt{(x^2-6)^8} \\ & = -\sqrt{[(x^2-6)^4]^2} \\ & = \boxed{-(x^2-6)^4} \end{aligned}$$

$$\begin{aligned} \text{c. } & \sqrt[5]{243a^{20}b^{25}} \\ & = \sqrt[5]{3^5(a^4)^5(b^5)^5} \\ & = \boxed{3a^4b^5} \end{aligned}$$

$$\begin{aligned} \text{d. } & \sqrt{-16x^4y^8} \\ & \text{no } \mathbb{R} \text{ roots} \\ & \text{since } \sqrt{-16} \text{ not} \\ & \text{a } \mathbb{R} \text{ \#} \end{aligned}$$

$$\begin{aligned} \text{e. } & \pm \sqrt{36x^{10}} \\ & = \pm \sqrt{6^2 (x^5)^2} \\ & = \boxed{\pm 6x^5} \end{aligned}$$

$$\begin{aligned} \text{f. } & -\sqrt{(y+7)^{16}} \\ & = -\sqrt{[(y+7)^8]^2} \\ & = \boxed{-(y+7)^8} \end{aligned}$$

$$\begin{aligned} \text{g. } & \sqrt[4]{y^4} \\ & = \sqrt[4]{(y)^4} \\ & = \boxed{|y|} \text{ * since } y \\ & \text{could be neg, use} \\ & \text{absolute value to identify} \\ & \text{principal root} \end{aligned}$$

$$\begin{aligned} \text{h. } & \sqrt[6]{64(x^2-3)^{18}} \\ & = \sqrt[6]{2^6 [(x^2-3)^3]^6} \\ & = \boxed{2 |(x^2-3)^3|} \end{aligned}$$

$$\begin{aligned} \text{i. } & \sqrt{36y^6} \\ & = \sqrt{6^2 (y^3)^2} \\ & = \boxed{6 |y^3|} \end{aligned}$$

$$\begin{aligned} \text{j. } & \sqrt[4]{16(x-3)^{12}} \\ & = \sqrt[4]{2^4 [(x-3)^3]^4} \\ & = \boxed{2 |(x-3)^3|} \end{aligned}$$

6.4 nth Roots
Honors Algebra 2

k. $\pm\sqrt{100y^8}$
 $\pm 10y^4$

n. $\sqrt[4]{16g^{16}h^{24}}$
 $= 2g^4h^6$

l. $-\sqrt{49u^8b^{12}}$
 $= -7u^4b^6$

o. $\sqrt{-16y^4}$
 $= \text{no solution}$

m. $\sqrt{(y-6)^8}$
 $= (y-6)^4$

p. $\sqrt[6]{64(2y+1)^{18}}$
 $= \sqrt[6]{2^6 [(2y+1)^3]^6}$
 $= 2 |(2y+1)^3|$

