1. Simplify:

a. $\sqrt{32x^8}$

$= \sqrt{2^5 x^8}$

$= \sqrt{2^2\ 2^2\ 2^1\ x^2 x^2 x^2 x^2}$

$= 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \sqrt{2}$

$= \boxed{4x^4\sqrt{2}}$

b. $\sqrt[4]{16a^{24}b^{13}}$

$= \sqrt[4]{2^4\ a^{24}\ b^{12}\ b}$

$= \boxed{2a^6 b^3 \sqrt[4]{b}}$

c. $\sqrt{12c^6d^3}$

$= \sqrt{4 \cdot 3\ c^6\ d^2\ d}$

$= \boxed{2c^3 d\sqrt{3d}}$

d. $\sqrt[3]{27y^{12}z^7}$

$= \sqrt[3]{3^3\ y^{12}\ z^6\ z}$

$= \boxed{3y^4 z^2 \sqrt[3]{z}}$

e. $\sqrt{36ab^4c^5}$

$= \sqrt{6^2 ab^4 c^4 c}$

$= \boxed{6b^2c^2\sqrt{ac}}$

f. $\frac{\sqrt{x^6}}{\sqrt{y^7}}$ *rationalize den.

$= \frac{\sqrt{x^6}}{\sqrt{y^7}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$

$= \frac{\sqrt{x^6 y}}{\sqrt{y^8}}$

$= \boxed{\frac{\sqrt{x^6 y}}{y^4}}$

g. $\sqrt[4]{\frac{6}{5x}}$

$= \frac{\sqrt[4]{6}}{\sqrt[4]{5x}} \cdot \frac{\sqrt[4]{5^3 x^3}}{\sqrt[4]{5^3 x^3}}$

$= \frac{\sqrt[4]{750x^3}}{\sqrt[4]{5^4 x^4}}$

$= \boxed{\frac{\sqrt[4]{750x^3}}{5x}}$

h. $\frac{\sqrt{a^3}}{\sqrt{b^5}} \cdot \frac{\sqrt{b}}{\sqrt{b}}$

$= \frac{\sqrt{a^9 b}}{\sqrt{b^6}}$

$= \boxed{\frac{a^4\sqrt{ab}}{b^3}}$

i. $\sqrt[5]{\frac{3}{4y}}$

$= \frac{\sqrt[5]{3}}{\sqrt[5]{4y}}$

$= \frac{\sqrt[5]{3}}{\sqrt[5]{4y}} \cdot \frac{\sqrt[5]{4^4 y^4}}{\sqrt[5]{4^4 y^4}}$

$= \boxed{\frac{\sqrt[5]{768 y^4}}{4y}}$

j. $\sqrt[4]{\frac{5x}{8y}}$

$= \frac{\sqrt[4]{5x}}{\sqrt[4]{8y}} \cdot \frac{\sqrt[4]{8^3 y^3}}{\sqrt[4]{8^3 y^3}}$

$= \boxed{\frac{\sqrt[4]{2560 x y^3}}{8y}}$

2. Simplify:

* can multiply / divide $\sqrt{}$ when index is same

$\sqrt{x} \cdot \sqrt[3]{y}$ can't simplify

a. $5\sqrt[3]{-12ab^4} \cdot 3\sqrt[3]{18a^2b^2}$

216 = $2^3 \cdot 3^3$

$= 15\sqrt[3]{-216 a^3 b^5}$

$= 15\sqrt[3]{-2^3 \cdot 3^3 a^3 b^3 b^2}$

$= 15(-2)(3)ab\sqrt[3]{b^2}$

$= \boxed{-90ab\sqrt[3]{b^2}}$

b. $6\sqrt{8c^3d^5} \cdot 4\sqrt{2cd^3}$

$= 24\sqrt{16 c^4 d^8}$

$= 24\sqrt{4^2 c^2 c^2 d^2 d^2 d^2 d^2}$

$= 24 \cdot 4 \cdot c \cdot c \cdot d \cdot d \cdot d \cdot d$

$= \boxed{96 c^2 d^4}$

c. $2\sqrt[4]{8x^3y^2} \cdot 3\sqrt[4]{2x^5y^2}$

$= 6\sqrt[4]{16x^8y^4}$

$= 6(2)x^2y$

$= \boxed{12x^2y}$

d. $3\sqrt[3]{36xy} \cdot 2\sqrt[3]{6x^2y^2}$

$= 6\sqrt[3]{216x^3y^3}$

$= 6\sqrt[3]{6^3x^3y^3}$

$= 6 \cdot 6xy$

$= \boxed{36xy}$

* can add / sub $\sqrt{\ }$ only if index and term inside $\sqrt{\ }$ the same *

3. Simplify:

a. $\sqrt{98} - 2\sqrt{32}$

98 → 2, 49 → 7, 7

$= \sqrt{7^2 \cdot 2} - 2\sqrt{16 \cdot 2}$

$= 7\sqrt{2} - 8\sqrt{2}$

$= \boxed{-\sqrt{2}}$

b. $4\sqrt{8} + 3\sqrt{50}$

$= 4\sqrt{4 \cdot 2} + 3\sqrt{25 \cdot 2}$

$= 4 \cdot 2\sqrt{2} + 3 \cdot 5\sqrt{2}$

$= 8\sqrt{2} + 15\sqrt{2}$

$= \boxed{23\sqrt{2}}$

c. $5\sqrt{12} + 2\sqrt{27} - \sqrt{128}$

128 → 2, 64 → 2, 32 → 2, 16 → 2, 8 → 2, 4 → 2, 2

$= 5\sqrt{4 \cdot 3} + 2\sqrt{9 \cdot 3} - \sqrt{2^7}$

$= 5 \cdot 2\sqrt{3} + 2 \cdot 3\sqrt{3} - 2^3\sqrt{2}$

$= 10\sqrt{3} + 6\sqrt{3} - 8\sqrt{2}$

$= \boxed{16\sqrt{3} - 8\sqrt{2}}$

4. Simplify:

a. $(4\sqrt{3} + 5\sqrt{2})(3\sqrt{2} - 6)$

First, outer, inner, last

$= 12\sqrt{6} - 24\sqrt{3} + 15\sqrt{4} - 30\sqrt{2}$

$= 12\sqrt{6} - 24\sqrt{3} + 30 - 30\sqrt{2}$

b. $(6\sqrt{3}-5)(2\sqrt{5}+4\sqrt{2})$

$= 12\sqrt{15} + 24\sqrt{6} - 10\sqrt{5} - 20\sqrt{2}$

c. $(7\sqrt{2}-3\sqrt{3})(7\sqrt{2}+3\sqrt{3})$ *conjugates

$= 49\sqrt{4} + 21\sqrt{6} - 21\sqrt{6} - 9\sqrt{9}$

$= 49(2) - 9(3)$

$= 98 - 27$

$= 71$

5. Simplify:

a. $\frac{5}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{10-5\sqrt{3}}{4-\sqrt{9}}$

$= \frac{10-5\sqrt{3}}{4-3}$

$= \boxed{10-5\sqrt{3}}$

b. $\frac{8}{\sqrt{6}-5} \cdot \frac{\sqrt{6}+5}{\sqrt{6}+5}$

$= \frac{8\sqrt{6}+40}{\sqrt{36}+5\sqrt{6}-5\sqrt{6}-25}$

$= \frac{8\sqrt{6}+40}{6-25}$

$= \frac{8\sqrt{6}+40}{-19} = \boxed{\frac{-8\sqrt{6}-40}{19}}$

c. $\frac{4+\sqrt{2}}{\sqrt{2}-3}$

$= \frac{4+\sqrt{2}}{\sqrt{2}-3} \cdot \frac{\sqrt{2}+3}{\sqrt{2}+3}$

$= \frac{4\sqrt{2}+12+\sqrt{4}+3\sqrt{2}}{\sqrt{4}-9}$

$= \frac{7\sqrt{2}+14}{-7}$

$= \boxed{-\sqrt{2}-2}$