

6.6 Rational Exponents

Honors Algebra 2

1. Write the following in radical or exponential form:

a. $x^{\frac{1}{6}}$

$$= \sqrt[6]{x^1}$$

c. $y^{\frac{5}{6}}$

$$= \sqrt[6]{y^5}$$

OR

$$= \sqrt[6]{y^5}$$

e. $\sqrt[3]{c^{-5}}$

$$= c^{-5/3}$$

b. $\sqrt[4]{w^3}$

$$= w^{3/4}$$

d. $(\sqrt[5]{t})^6$

$$= t^{6/5}$$

f. $p^{\frac{3}{2}}$

$$= \sqrt{p^3}$$

OR

$$= \sqrt{p^3}$$

2. Simplify the following:

a. $27^{\frac{2}{3}}$

$$= \sqrt[3]{27^2}$$

$$= 3^2$$

$$= \boxed{9}$$

d. $125^{\frac{2}{3}}$

$$= \sqrt[3]{125^2}$$

$$= 5^2$$

$$= \boxed{25}$$

b. $(-16)^{\frac{3}{2}}$

$$= \sqrt{-16^3}$$

$$= \boxed{\text{No Solution}}$$

e. $\frac{24}{4^{\frac{3}{2}}}$

$$= \frac{24}{\sqrt{4^3}}$$

$$= \frac{24}{2^3}$$

$$= \frac{24}{8}$$

$$= \boxed{3}$$

c. $32^{-\frac{1}{5}}$

$$= \frac{1}{32^{1/5}}$$

$$= \frac{1}{\sqrt[5]{32}}$$

$$= \boxed{\frac{1}{2}}$$

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3. Simplify the following expressions:

a. $a^{2/7} \cdot a^{4/7}$

$$= a^{2/7 + 4/7}$$

$$= a^{6/7}$$

b. $p^{1/4} \cdot p^{9/4}$

$$= p^{1/4 + 9/4}$$

$$= p^{10/4}$$

$$= p^{5/2}$$

c. $r^{-4/5}$

$$= \frac{1}{r^{4/5}}$$

d. $a^{3/4} \cdot a^{1/2}$

$$= a^{3/4 + 1/2}$$

$$= a^{3/4 + 2/4}$$

$$= a^{5/4}$$

e. $\frac{x^{4/5}}{x^{1/5}}$

$$= x^{4/5 - 1/5}$$

$$= x^{3/5}$$

f. $\frac{b^3}{c^{1/2}} \cdot \frac{c}{b^{1/3}}$

$$= b^{3 - 1/3} c^{1 - 1/2}$$

$$= b^{9/3 - 1/3} c^{1/2}$$

$$= b^{8/3} c^{1/2}$$

4. Simplify each expression:

a. $\frac{\sqrt[4]{27}}{\sqrt{3}}$

$$= \frac{27^{1/4}}{3^{1/2}}$$

$$= \frac{(3^3)^{1/4}}{3^{1/2}}$$

$$= \frac{3^{3/4}}{3^{1/2}}$$

$$= 3^{3/4 - 1/2}$$

$$= 3^{3/4 - 2/4}$$

$$= 3^{1/4}$$

b. $\sqrt[3]{64z^6}$

$$= \sqrt[3]{4^3 z^6}$$

$$= 4z^2$$

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$$\begin{aligned}
 \text{c. } & \frac{x^{1/2}-2}{3x^{1/2}+2} \cdot \frac{3x^{1/2}-2}{3x^{1/2}-2} \\
 = & \frac{(x^{1/2}-2)(3x^{1/2}-2)}{9x-4} \\
 = & \frac{3x-2\sqrt{x}-6\sqrt{x}+4}{9x-4} = \boxed{\frac{3x-8\sqrt{x}+4}{9x-4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } & \frac{\sqrt[3]{32}}{\sqrt[3]{2}} \\
 = & \frac{32^{1/4}}{2^{1/2}} \\
 = & \frac{(2^5)^{1/4}}{2^{1/2}} \\
 = & \frac{2^{5/4}}{2^{1/2}} \\
 = & 2^{5/4-1/2} \\
 = & 2^{3/4} = \boxed{2^{3/4}}
 \end{aligned}$$

$$\begin{aligned}
 \text{e. } & \sqrt[3]{16x^4} \\
 = & \sqrt[3]{2^4 x^4} \\
 = & \boxed{2x \sqrt[3]{2x}}
 \end{aligned}$$

$$\begin{aligned}
 \text{f. } & \frac{y^{1/2}+2}{y^{1/2}-2} \cdot \frac{y^{1/2}+2}{y^{1/2}+2} \\
 = & \frac{(y^{1/2}+2)(y^{1/2}+2)}{y-4} \\
 = & \frac{y+4y^{1/2}+4}{y-4} \\
 = & \boxed{\frac{y+4\sqrt{y}+4}{y-4}}
 \end{aligned}$$

ConceptSummary Expressions with Rational Exponents

An expression with rational exponents is simplified when all of the following conditions are met.

- It has no negative exponents.
- It has no exponents that are not positive integers in the denominator.
- It is not a complex fraction.
- The index of any remaining radical is the least number possible.

