6.7 Solving Equations with nth Roots and
Solving Radical Equations
Honors Algebra 2

1. Solve the following equations:

a. $-2x^6 = -1458$

$x^6 = 729$

$x = \pm\sqrt[6]{729}$

$x = \pm\sqrt[6]{3^6}$

$x = \pm 3$

729 = 3 · 243 = 3 · 3 · 81 = 3 · 3 · 9 · 9 = 3 · 3 · 3 · 3 · 3 · 3

b. $x^3 - 9 = 31$

$x^3 = 40$

$x = \sqrt[3]{40}$

no ±

$x = \sqrt[3]{2^3 \cdot 5}$

$x = 2\sqrt[3]{5}$

40 = 4 · 10 = 2 · 2 · 5 · 2

c. $12 - (x+3)^3 = 84$

$-(x+3)^3 = 72$

$(x+3)^3 = -72$

$x+3 = \sqrt[3]{-72}$

$x+3 = \sqrt[3]{-1 \cdot 3^2 \cdot 2^3}$

$x+3 = -2\sqrt[3]{3^2}$

$x = -3 - 2\sqrt[3]{9}$

72 = 2 · 36 = 2 · 2 · 18 = 2 · 2 · 2 · 9 = 2 · 2 · 2 · 3 · 3

Radical Equation: an equation with radicals that have variables in their radicands. Solve within the set of REAL NUMBERS!

Case #1: The variable is not under the radical and is in two separate terms.

- → Move the terms with the variable to one side
- → Factor out x
- → Divide by the Coefficient
- → Rationalize the denominator
- → Check Solutions

2. $x + 1 = x\sqrt{2}$

$1 = x\sqrt{2} - x$

$1 = x(\sqrt{2} - 1)$

$\frac{1}{\sqrt{2}-1} = x$

$\frac{1(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = x$

$\frac{\sqrt{2}+1}{2-1} = x$

$\sqrt{2}+1 = x$

3. $x\sqrt{3} = 3x + 5$

$x\sqrt{3} - 3x = 5$

$x(\sqrt{3} - 3) = 5$

$x = \frac{5}{\sqrt{3}-3}$

$x = \frac{5(\sqrt{3}+3)}{(\sqrt{3}-3)(\sqrt{3}+3)}$

$x = \frac{5\sqrt{3}+15}{3-9}$

$x = \frac{5\sqrt{3}+15}{-6}$

$x = \frac{-5\sqrt{3}-15}{6}$

Case #2: One variable and is IN the radical

- → Isolate the term with the radical
- → Undo the radical operation - INVERSE!
- → Solve for the variable
- → Check the solutions

4. $\sqrt{2b+2} - 3 = -15$

$\sqrt{2b+2} = -12$

$(\sqrt{2b+2})^2 = (-12)^2$

$2b + 2 = 144$

$2b = 142$

$b = 71$

check:

$\sqrt{2(71)+2} - 3 = -15$

$\sqrt{144} - 3 = -15$

$12 - 3 = -15$

$9 = -15$

extraneous solution

No solution

5. $(x-1)^{2/3} + 2 = 6$

$(x-1)^{2/3} = 4$

$\left[(x-1)^{2/3}\right]^{3/2} = 4^{3/2}$

$x - 1 = 4^{3/2}$

$x - 1 = \sqrt{4}^3$

$x - 1 = 2^3$

$x - 1 = 8$

$x = 9$

check:

$(9-1)^{2/3} + 2 = 6$

$8^{2/3} + 2 = 6$

$\sqrt[3]{8}^2 + 2 = 6$

$2^2 + 2 = 6$

$6 = 6$ ✓

$x = 9$

6. $x^4 + 81 = 0$

$x^4 = -81$

$x = \pm\sqrt[4]{-81}$

neg → not possible

No ℝ solution

Case #3: The variable is IN TWO separate radicals

- → Separate the radical terms to opposite sides of the equation
- → Undo the radical operation (square both sides)
- → Isolate the remaining radical term
- → Undo the radical operation - INVERSE!
- → Solve for the variable
- → Check Solutions

7. $\sqrt{x+1} - \sqrt{3x} = -1$

$\sqrt{x+1} = -1 + \sqrt{3x}$

$(\sqrt{x+1})^2 = (-1+\sqrt{3x})^2$

$x+1 = (-1+\sqrt{3x})(-1+\sqrt{3x})$

$x+1 = 1 - \sqrt{3x} - \sqrt{3x} + 3x$

$x+1 = 1 - 2\sqrt{3x} + 3x$

completely isolate radical!

$-2x = -2\sqrt{3x}$

$x = \sqrt{3x}$

$x^2 = (\sqrt{3x})^2$

$x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = \cancel{0}, 3$ (extraneous)

check:

$\sqrt{0+1} - \sqrt{3(0)} = -1$

$1 = -1$

No

$\sqrt{3+1} - \sqrt{9} = -1$

$2 - 3 = -1$

$-1 = -1$

yes

8. $\sqrt{x+10} = 8 - \sqrt{x-6}$

$(\sqrt{x+10})^2 = (8 - \sqrt{x-6})^2$

$x+10 = 64 - 16\sqrt{x-6} + (x-6)$

$x+10 = 58 + x - 16\sqrt{x-6}$

$-48 = -16\sqrt{x-6}$

$3 = \sqrt{x-6}$

$3^2 = (\sqrt{x-6})^2$

$9 = x-6$

$15 = x$

check:

$\sqrt{15+10} = 8 - \sqrt{15-6}$

$5 = 8 - \sqrt{9}$

$5 = 8 - 3$

$5 = 5$ ✓