Evaluate the following logarithms:

1. $\log_3 3 =$ 1	2. $\log_3 1 =$ 0
3. $\log_5 5 =$ 1	4. $\log_5 1 =$ 0
5. $\log 10 =$ 1	6. $\log 1 =$ 0
$\log_b b =$ 1	$\log_b 1 =$ 0

Properties of Logarithms

Condensed Form:	Expanded Form:
$\log_b m \cdot n$	$\log_b m + \log_b n$
$\log_b \frac{m}{n}$	$\log_b m - \log_b n$
$\log_b m^p$	$p \log_b m$

Expand the following expressions:

7. $\log_3(6 \cdot 2)$ $= \log_3 6 + \log_3 2$	8. $\log_7 3xy$ $= \log_7 3 + \log_7 x + \log_7 y$	9. $\log_3 3x^4$ $= \log_3 3 + \log_3 x^4$ $= \log_3 3 + 4\log_3 x$ $= 1 + 4\log_3 x$
10. $\log_6 \frac{5}{y}$ $= \log_6 5 - \log_6 y$	11. $\log \sqrt[5]{2x}$ $= \log (2x)^{1/5}$ $= \frac{1}{5} \log (2x)$ $= \frac{1}{5}[\log 2 + \log x]$ $= \frac{1}{5}\log 2 + \frac{1}{5}\log x$	12. $\log_2 \sqrt{x}$ $= \log_2 x^{1/2}$ $= \frac{1}{2}\log_2 x$

Developing Intuition for Logarithms*

Condense the following expressions:

13. $\log 9 + 3\log 2 - \log 3$ $= \log 9 + \log 2^3 - \log 3$ $= \log(9 \cdot 2^3) - \log 3$ $= \log\left(\frac{9 \cdot 2^3}{3}\right)$ $= \log(72/3)$ $= \log 24$	14. $\log 4 + 3\log 3 - \log 12$ $= \log 4 + \log 3^3 - \log 12$ $= \log(4 \cdot 3^3) - \log 12$ $= \log\left(\frac{4 \cdot 3^3}{12}\right)$ $= \log 9$	15. $\frac{1}{2}\log_3 64 + \log_3 x$ $\log_3 \sqrt{64} + \log_3 x$ $\log_3 8 + \log_3 x$ $\log_3 8x$
16. $2\log 8 - \log 4 - \log 16$ $\log 8^2 - \log 4 - \log 16$ $\log 64/4 - \log 16$ $\log 16 - \log 16$ $\log 1$ 0	17. $4\log x - 6\log 2$ $\log x^4 - \log 2^6$ $\log \frac{x^4}{64}$	18. $2(\log_3 12 - \log_3 3) + \frac{1}{3}\log_3 8$ $2(\log_3 12/3) + \frac{1}{3}\log_3 8$ $\log_3 4^2 + \log_3 \sqrt[3]{8}$ $\log_3 16 + \log_3 2$ $\log_3 32$

For the following examples use $\log_3 2 \approx 0.631$ and $\log_3 7 \approx 1.7712$ to evaluate the logarithm using the properties of logarithms. Do not use a calculator!

19. $\log_3 4$ $= \log_3(2 \cdot 2)$ $= \log_3 2 + \log_3 2$ $= 0.631 + 0.631$ $= 1.262$	20. $\log_3 \frac{7}{2}$ $= \log_3 7 - \log_3 2$ $= 1.7712 - 0.631$ $= 1.1402$	21. $\log_3 108$ $= \log_3(2^2 \cdot 3^3)$ $= \log_3 2^2 + \log_3 3^3$ $= 2\log_3 2 + 3$ $= 1.262 + 3$ $= 4.262$

108
2 54
2 27
3 9 3 3

Solve the following using the Properties of Logarithms:

22. $\log_3 2 - \log_3 6 = \log_3 x$

$\log_3 2/6 = \log_3 x$

$\frac{1}{3} = x$

23. $2\log_3 y + \log_3 0.1 = \log_3 5 + \log_3 2$

$\log_3 y^2 + \log_3 0.1 = \log_3 (5 \cdot 2)$

$\log_3 0.1y^2 = \log_3 10$

$0.1y^2 = 10$

$\frac{1}{10} y^2 = 10$

$y^2 = 100$

$y = \pm 10$ -10 extraneous

$\boxed{y = 10}$

24. $\log_2(x - 3) = \log_2 10 - \log_2 \frac{5}{2} + \log_2 \frac{1}{2}$

$\log_2 (x-3) = \log_2 (10 \div 5/2) + \log_2 1/2$

$\log_2 (x-3) = \log_2 4 + \log_2 1/2$

$\log_2 (x-3) = \log_2 4 \cdot 1/2$

$\log_2 (x-3) = \log_2 2$

$x - 3 = 2$

$x = 5$

25. $\log_2 x + \log_2(x - 4) = 5$

$\log_2 x(x-4) = 5$

$2^5 = x(x-4)$

$32 = x^2 - 4x$

$0 = x^2 - 4x - 32$

$0 = (x-8)(x+4)$

$x = 8, \cancel{-4}$

$\boxed{x = 8}$