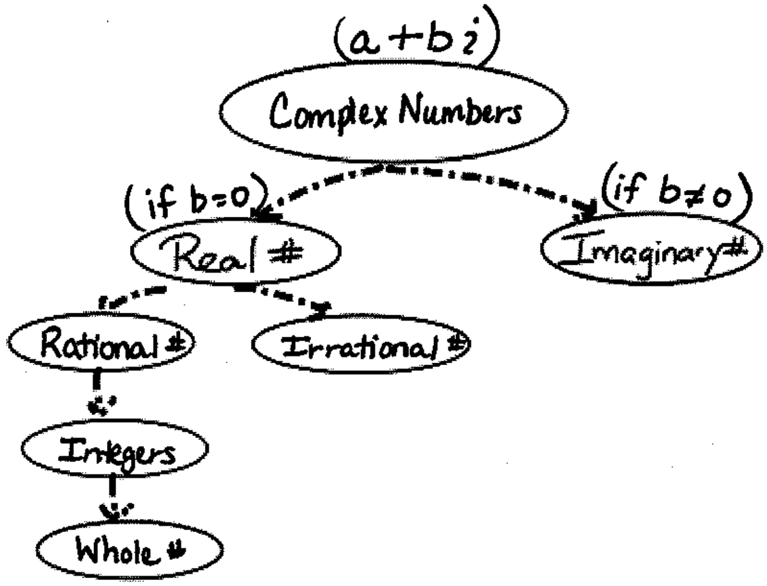
Honors Algebra 2 Name Key

4.6 Perform Operations with Complex Numbers

How do you solve: $x^2 + 16 = 0$? $\rightarrow$ $x^2 = -16$

$x = \pm\sqrt{-16}$

But there isn't any Real # that we can square to get −16! There must be some kind of number that will get us −16... IMAGINARY #



The **imaginary unit**, i, can be used to describe the square roots of negative numbers.

$i = \sqrt{-1}$

$i^2 = \sqrt{-1}^{\,2} \rightarrow i^2 = -1$

When there is a negative radicand under a square root, you must take out i before simplifying or performing operations!

Simplify:

1. $\sqrt{-9} = \sqrt{-1}\cdot\sqrt{9}$ $= i \cdot 3$ $= 3i$	2. $\sqrt{-12} = i\sqrt{12}$ $= i\sqrt{4}\cdot\sqrt{3}$ $= 2i\sqrt{3}$	3. $\sqrt{-72} = i\sqrt{72}$ $= i\sqrt{36}\cdot\sqrt{2}$ $= 6i\sqrt{2}$
4. $-3i \cdot 8i = -24i^2$ $= -24(-1)$ $= 24$	5. $\sqrt{-6}\cdot\sqrt{-10}$ $= i\sqrt{6}\cdot i\sqrt{10}$ $= i^2\sqrt{60}$ $= -1\sqrt{4}\cdot\sqrt{15}$ $= -2\sqrt{15}$	6. $(3\sqrt{-5})^2$ $= (3i\sqrt{5})^2$ $= 9i^2 \cdot \sqrt{25}$ $= -45$

Solve.

7. $4x^2 + 36 = 0$ $4x^2 = -36$ $x^2 = -9$ $x = \pm\sqrt{-9}$ $x = \pm 3i$ 2 imaginary roots!	8. $3x^2 + 40 = -x^2 - 56$ $4x^2 = -96$ $x^2 = -24$ $x = \pm\sqrt{24}$ $x = \pm i\sqrt{4}\cdot\sqrt{6}$ $x = \pm 2i\sqrt{6}$	9. $(x-3)^2 = -12$ take $\sqrt{\ }$ Now! $x - 3 = \pm\sqrt{-12}$ $x - 3 = \pm 2i\sqrt{3}$ $x = 3 \pm 2i\sqrt{3}$

A **complex number** is a number that can be written in the form a + b*i*, where a & b are real numbers and *i* is the imaginary unit.

- **a** is called the real part and **b*i*** is called the imaginary part of the complex number.
- When writing complex numbers, the real part should be written first (standard form). $a+bi$
- When adding of subtracting complex numbers, combine the real parts and the imaginary parts. For example, (a + bi) – (c + di) = (a - c) + (b – d)i.

Simplify.

10. $(8 - 5i) + (2 + i)$

$(8+2)+(-5i+i)$

$10-4i$

11. $(4 + 7i) - (2 + 3i)$

$(4-2)+(7i-3i)$

$2+4i$

12. $(4 + 2i)(3 - 5i)$

$12-20i+6i-10i^2$

$12-14i+10$

$22-14i$

When dividing, an imaginary number cannot be left in the denominator (remember *i* is the square root of -1 and we have a rule to not leave any roots in a denominator). **To eliminate the imaginary number in the denominator, multiply by the complex conjugate.**

The complex conjugate of a + bi is: $a-bi$.

Simplify.

13. $\frac{5}{1+i}\cdot\frac{1-i}{1-i}$

$=\frac{5-5i}{1\cancel{-i+i}-i^2}$

$=\frac{5-5i}{1+1}$

$=\frac{5-5i}{2}$

$=\frac{5}{2}-\frac{5}{2}i$

14. $\frac{5+2i}{3-2i}\cdot\frac{3+2i}{3+2i}$

$=\frac{15+10i+6i+\cancel{4i^2}-4}{9+\cancel{6i}-\cancel{6i}\cancel{-4i^2}+4}$

$=\frac{11+16i}{13}$

$=\frac{11}{13}+\frac{16}{13}i$

15. $\frac{2+i}{3i}\cdot\frac{3i}{3i}$ *or just multiply by

$=\frac{6i+3i^2}{9i^2}$

$=\frac{-3+6i}{-9}$

$=\frac{-3}{-9}+\frac{6}{-9}i$

$=\frac{1}{3}-\frac{2}{3}i$