Key

Midterm Review - Honors Algebra 2

Part I No Calculator

1. Line 1 contains (2, – 4) and (0, 2). Line 2 contains (– 4, 5) and (– 1, 6). Are the lines parallel, perpendicular, or neither?

$m_1 = \frac{-4-2}{2-0} = \frac{-6}{2} = -3$

$m_2 = \frac{5-6}{-4+1} = \frac{-1}{-3} = \frac{1}{3}$

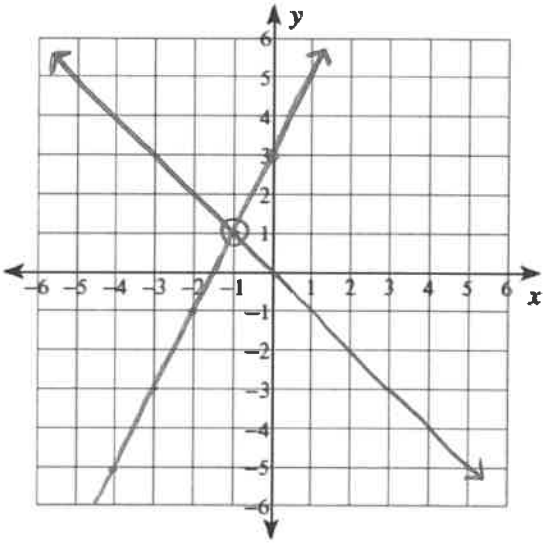
perpendicular

2. Graph each line. Determine the slope, and x- and y-intercepts for each function. Determine the solution to the system.

$y = -x$
$y = 2x + 3$

$y = -x$
$m = -1$
y-int (0,0)
x-int (0,0)

$y = 2x + 3$
$m = 2$
y-int (0,3)
x-int $(-\frac{3}{2}, 0)$



solution = (-1, 1)

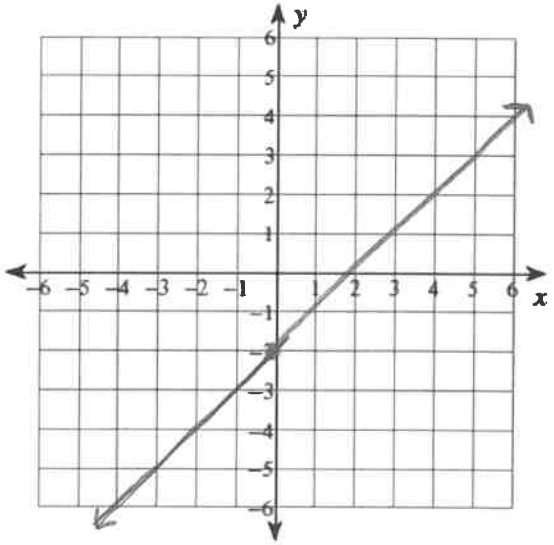
3. Graph each line. Determine the slope, and x- and y-intercepts for each function. Determine the solution to the system.

$-x + y = -2$
$2x - 2y = 4$

$y = x - 2$

$-2y = -2x + 4$
$y = x - 2$

They are same line
$m = 1$
y-int (0, -2)
x-int (2, 0)



solution = ∞

4. Solve the system using any algebraic method:

a) $5x-2y=-7$
$-3x+2y=5$

$2x = -2$

$x = -1$

$5(-1) - 2y = -7$

$-2y = -2$ $\quad y = 1$

$(-1, 1)$

b) $6x+3y=-3$
$4x-4y=-8$

$x = y - 2$

$6(y-2) + 3y = -3$

$6y - 12 + 3y = -3$

$9y = 9$

$y = 1$

$x = 1 - 2$

$= -1$

$(-1, 1)$

5. Graph the system of inequalities:

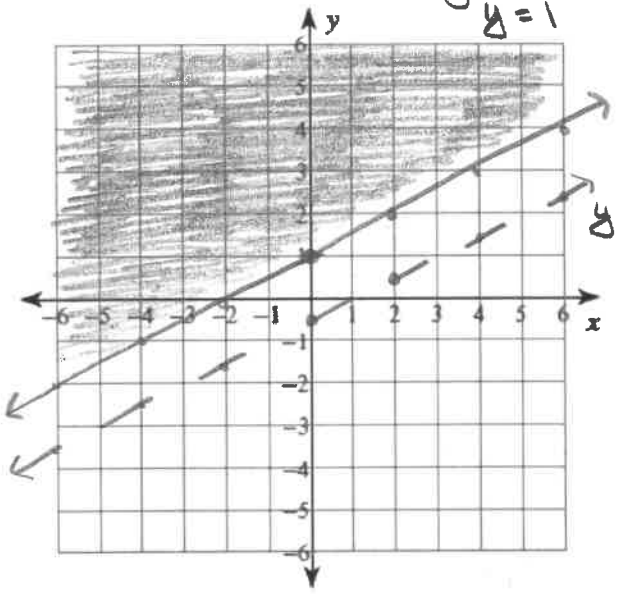
$x-2y \leq -2$
$2x-4y < 2$

$-2y \leq -x - 2$

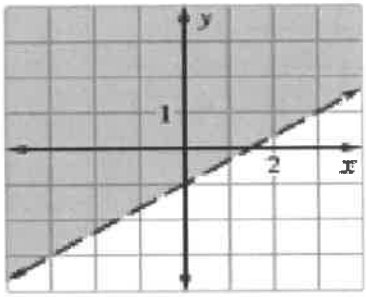
$y \geq \frac{1}{2}x + 1$

$-4y < -2x + 2$

$y > \frac{1}{2}x - \frac{1}{2}$



6. Write an inequality represented by the graph.



$b = -1 \quad m = 2/3$

$y > \frac{2}{3}x - 1$

Perform the indicated

7) $\begin{bmatrix} -2 & 0 & 7 \\ 11 & -3 & -5 \end{bmatrix} + \begin{bmatrix} -10 & 4 & 8 \\ 1 & -6 & 6 \end{bmatrix}$

$\begin{bmatrix} -12 & 4 & 15 \\ 12 & -9 & 1 \end{bmatrix}$

8) $2\begin{bmatrix} -1 & 2 & 3 \\ 3 & 0 & -4 \end{bmatrix}$

$\begin{bmatrix} -2 & 4 & 6 \\ 6 & 0 & -8 \end{bmatrix}$

9) $\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 & 0 \\ 1 & -4 & -1 \end{bmatrix}$

$1\times 2 \quad 2\times 3 \rightarrow 1\times 3$

$= \begin{bmatrix} (6+1) & (-4-4) & (0-1) \end{bmatrix}$

$= [7, -8, -1]$

10) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & 0 \end{bmatrix}$

$2\times 2 \quad 2\times 3 \rightarrow 2\times 3$

$= \begin{bmatrix} 1-2 & 2-2 & 3+0 \\ 0-1 & 0-1 & 0+0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 3 \\ -1 & -1 & 0 \end{bmatrix}$

Evaluate the determinant of the matrix

11) $\begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix}$

$(2)(-2) - (4)(-1)$

$-4 + 4$

$\boxed{0}$

12) $\begin{bmatrix} 1 & 3 & -2 \\ 3 & -1 & -6 \\ 4 & 2 & -8 \end{bmatrix}$

$\begin{bmatrix} (1)(-1)(-8) + (3)(-6)(4) + (-2)(3)(2) \end{bmatrix} -$

$\begin{bmatrix} (-2)(-1)(4) + (1)(-6)(2) + (3)(3)(-8) \end{bmatrix}$

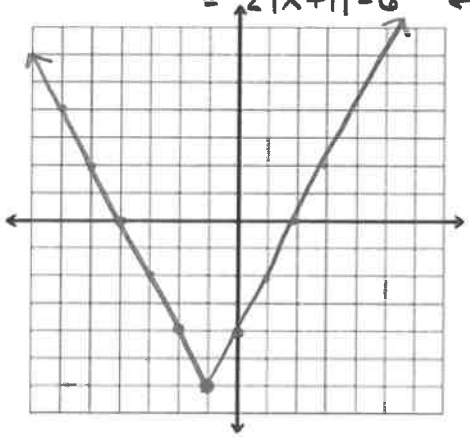
$= 0$

13. Graph the function:

a. $f(x) = |2x + 2| - 6$

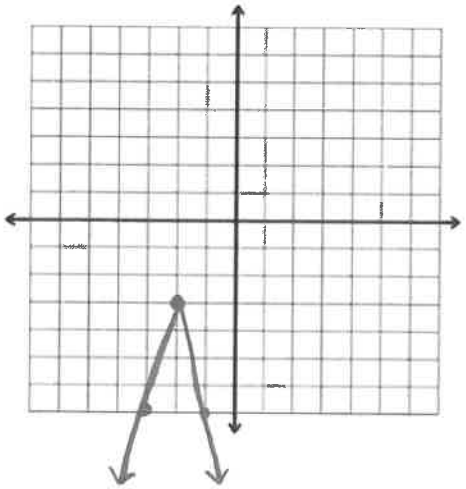
$f(x) = |2(x+1)| - 6 \quad \downarrow 6$

$= 2|x+1| - 6 \quad \leftarrow 1$



b. $y = -4|x + 2| - 3$

$\leftarrow 2 \quad \downarrow 3$



★ problem given as $f(x) = |2x|$ would be on test, but problem given as $f(x) = 2|x|$ could be on test

14. Tell whether the function opens up or down. Whether it has a maximum or a minimum value. What the maximum/minimum value. The coordinates of the vertex. The equation of the axis of symmetry. Write the equation in vertex form. Graph the parabola, be sure to plot five or more points.

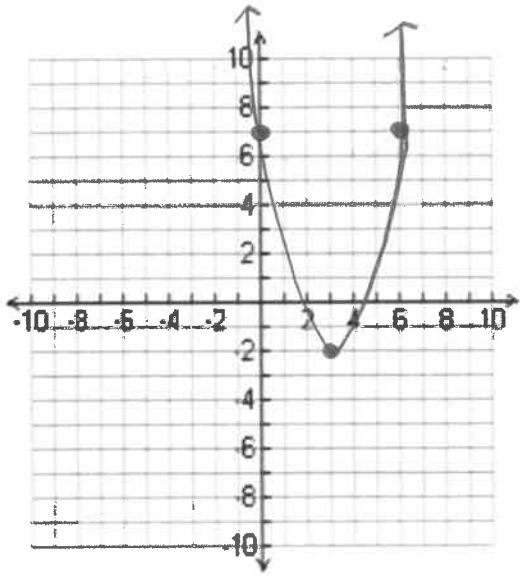
$y = x^2 - 6x + 7$

$\frac{-(-6)}{2(1)} = 3$

$f(3) = 3^2 - 6(3) + 7$

$= 9 - 18 + 7$

$= -2$



a) up or down: up

b) max or min: min

c) max or min value: (3, -2)

d) vertex: (3, -2)

e) axis of symmetry: $x = 3$

f) Vertex form: $y = (x-3)^2 - 2$

$y = x^2 - 6x + 7$

complete the square

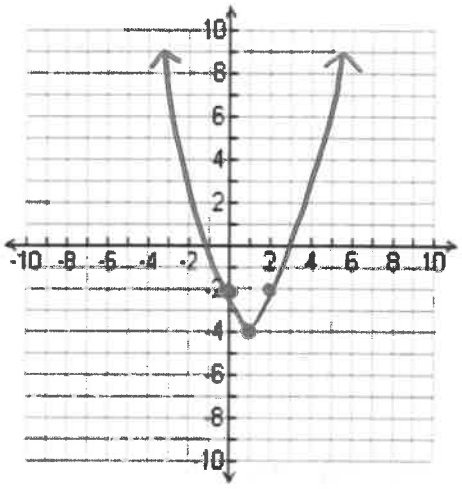
$y = (x^2 - 6x + 9) + 7 - 9$

$y = (x-3)^2 - 2$

15. Graph the function. Label the vertex and axis of symmetry. Then write in Standard form.

$y=2(x-1)^2-4$

d)



$y = 2(x^2-2x+1)-4$

$y = 2x^2-4x+2-4$

$y = 2x^2-4x-2$

a) vertex: $(1, -4)$

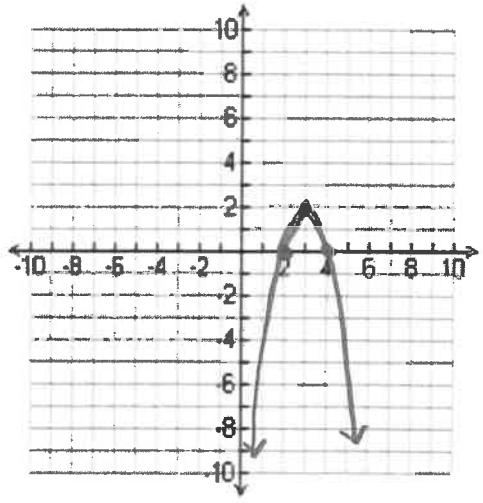
b) axis of symmetry: $x = 1$

d) Standard form: $y = 2x^2-4x-2$

16. For the parabola, tell where it crosses the x-axis; find the coordinates of the vertex; find the axis of symmetry; and write the equation in standard form. Then graph the parabola.

$y=-2(x-4)(x-2)$

e)



$x = 3$

$f(3) = -2(3-4)(3-2)$

$= -2(-1)(1)$

$= 2$

a) Intercepts: $(4, 0)$ $(2, 0)$

b) vertex: $(3, 2)$

c) axis of symmetry: $x = 3$

d) Standard form: $y = -2x^2+12x-16$

$y = -2(x^2-6x+8)$

$y = -2x^2+12x-16$

17. Write a quadratic function whose graph has the given characteristics. Vertex: (2, 3); point on graph: (0, – 1)

$y = a(x-h)^2 + k$

$y = a(x-2)^2 + 3$

$-1 = a(0-2)^2 + 3$

$-4 = 4a$

$-1 = a$

$\boxed{y = -1(x-2)^2 + 3}$

18. Write a quadratic function whose graph has the given characteristics. x-intercepts: – 3 & 1; point on graph: (2, 20)

$y = a(x+3)(x-1)$

$20 = a(2+3)(2-1)$

$20 = a(5)(1)$

$4 = a$

$\boxed{y = 4(x+3)(x-1)}$

19. Simplify:

a) $\frac{3}{2-\sqrt{5}}$

b) $\frac{2+\sqrt{2}}{1+\sqrt{2}}$

c) $(-2+14i)-(6-2i)$

$= \boxed{-8+16i}$

d) $(4-i)(8+3i)$

$= 32 + 12i - 8i - 3i^2$

$= 32 + 4i + 3$

$= \boxed{35 + 4i}$

e) $\frac{4i}{1-3i} \cdot \frac{1+3i}{1+3i}$

$= \frac{4i + 12i^2}{1 - 9i^2}$

$= \frac{-12+4i}{10} = \boxed{-\frac{6}{5} + \frac{2}{5}i}$

f) $\frac{7-4i}{2+3i} \quad \frac{2-3i}{2-3i}$

$= \frac{14 - 8i - 21i + 12i^2}{4 - 9i^2}$

$= \frac{14 - 29i - 12}{13}$

$= \boxed{\frac{2}{13} - \frac{29}{13}i}$

20. Factor completely:

a) $2x^2 + 5x - 12$

$(2x-3)(x+4)$

b) $4x^2 - 20x + 25$

$(2x-5)(2x-5)$

$(2x-5)^2$

c) $16a^2 - 81$

$(4a-9)(4a+9)$

$(2a)^3 + 5^3$

d) $8a^3 + 125$

$(2a+5)(4a^2 - 10a + 25)$

$(7m)^3 + (4n)^3$

e) $343m^3 + 64n^3$

$(7m+4n)(49m^2 - 28mn + 16n^2)$

f) $(m^3 - m^2) + (2m - 2)$

$m^2(m-1) + 2(m-1)$

$(m^2+2)(m-1)$

g) $(5n^3 - 10n^2) + (3n - 6)$

$5n^2(n-2) + 3(n-2)$

$(5n^2+3)(n-2)$

h) $x^4 - 7x^2 + 10$

$(x^2 - 5)(x^2 - 2)$

i) $5x^4 - 9x^2 + 4$

$(5x^2 - 4)(x^2 - 1)$

$(5x^2 - 4)(x+1)(x-1)$

j) $x^6 - 9x^3 + 8$

$(x^3 - 8)(x^3 - 1)$

$(x-2)(x^2+2x+4)(x-1)(x^2+x+1)$

k) $x^6 - 81$

$(x^2)^3 - 3^3$

$(x^2-3)(x^4 + 3x^2 + 9)$

21. Determine the discriminant. Then use it to determine the nature and number of solutions, then use the quadratic formula to solve the equation. Be sure to show ALL your work.

a) $x^2 + 6x = -15$

$x^2 + 6x + 15 = 0$

$b^2 - 4ac$

$6^2 - 4(1)(15)$

$36 - 60$

-24

2 Nonreal complex Root

b) $25x^2 - 17x = 13x - 9$

$25x^2 - 30x + 9 = 0$

$(30)^2 - 4(25)(9)$

$900 - 900$

0

1 Rational Root

22. Find each product.

a) $3n^2(4n^2 - 2n + 10)$

$= 12n^4 - 6n^3 + 30n^2$

b) $(2n+3)(n-2)$

$= 2n^2 - 4n + 6n - 6$

$= 2n^2 + 2n - 6$

c) $(-4x^2 - 5x - 1)(4x^2 - 6x - 2)$

$= -16x^4 + 24x^3 + 8x^2 - 20x^3 + 30x^2 + 10x - 4x^2 + 6x + 2$

$= -16x^4 + 4x^3 + 34x^2 + 16x + 2$

23. A polynomial f and one zero of f are given. Find the other zeros of f.

a) $f(x)=x^3+2x^2-20x+24;-6$

$$\begin{array}{c|cccc} -6 & 1 & 2 & -20 & 24 \\ & & -6 & 24 & -24 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$f(x) = (x+6)(x^2-4x+4)$

$f(x) = (x+6)(x-2)(x-2)$

$x=-6, 2$

b) $f(x)=15x^3-119x^2-10x+16;8$

$$\begin{array}{c|cccc} 8 & 15 & -119 & -10 & 16 \\ & & 120 & 8 & -16 \\ \hline & 15 & 1 & -2 & 0 \end{array}$$

$f(x) = (x-8)(15x^2+x-2)$

$= (x-8)(5x+2)(3x-1)$

$x = 8, -2/5, 1/3$

24. Solve by factoring

a) $3a^2+5a-28=0$

$(3a-7)(a+4)=0$

$a = 7/3, -4$

b) $3x(x^2-4)=0$

$3x(x+2)(x-2)=0$

$x=0, \pm 2$

25. Solve by using square roots.

a) $3(x-3)^2+2=26$

$3(x-3)^2 = 24$

$(x-3)^2 = 8$

$x-3 = \pm\sqrt{8}$

$x = 3 \pm 2\sqrt{2}$

b) $5(r-2)^2=35$

$(r-2)^2 = 7$

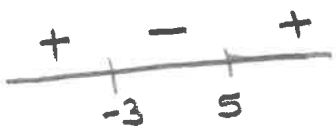
$r-2 = \pm\sqrt{7}$

$r = 2 \pm \sqrt{7}$

26. Solve the inequality $x^2 - 2x - 15 > 0$.

$(x-5)(x+3) > 0$

$x = 5, -3$



$(-\infty, -3) \cup (5, \infty)$

27. Find the solution set of the inequality $x^2 - 36 \leq 0$

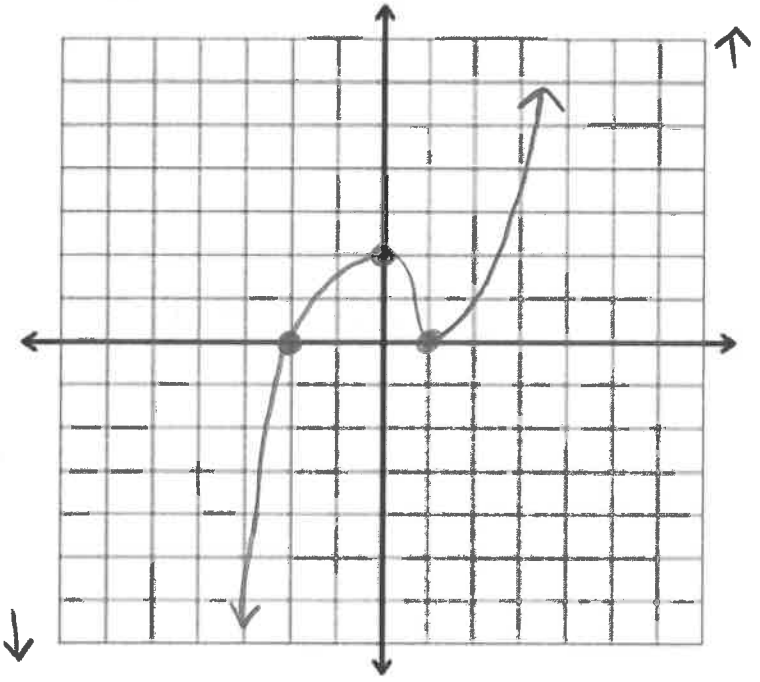
$(x-6)(x+6) \leq 0$

$x = \pm 6$

+	−	+
	−6	6

$[-6, 6]$

28. Graph the function $f(x) = (x + 2)(x - 1)^2$



x-int: $x = -2, 1$

crosses at -2

bounces at 1

end behavior: ↑ ↓

y-int: $y = (0+2)(0-1)^2$

$y = 2$

Part II Calculator

29. Write each polynomial in standard form. Name the leading coefficient, degree, number of terms, & name. don't have on midterm

a) $-10n+9-3n^3+n^2$

$-3n^3+n^2-10n+9$

leading coefficient = -3

degree = 3

# of terms = 4

cubic polynomial

b) $-8r^2-10r^4$

$-10r^4-8r^2$

lead. coeff = -10

degree = 4

2 terms

quartic binomial

c) $-9x-10+7x^2$

$7x^2-9x-10$

lead. coeff = 7

degree = 2

3 terms

quadratic trinomial

30. Find the local max and mins.

a) $f(x)=x^4-4x^3+2x^2+x+4$

min: (0.164, 3.908)

(2.574, −4.494)

max: (0.591, 4.586)

b) $f(x)=x^3+11x^2+35x+32$

min: (−2.333, −2.481)

max: (−5, 7)

31. Solve the system

$2x-y+z=-5$

$5x+2y-2z=19$

$x-3y+z=-5$

use calc or do out by hand

calc:

$$\begin{bmatrix} 2 & -1 & 1 \\ 5 & 2 & -2 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 19 \\ -5 \end{bmatrix}$$

A X B

$X = A^{-1}B$

$$X = \begin{bmatrix} 1 \\ -1/2 \\ -15/2 \end{bmatrix}$$

$x=1$

$y=-1/2$

$z=-15/2$

$\boxed{(1, -1/2, -15/2)}$

32. Solve

a) $\frac{1}{4}(\frac{4}{3}x+16)=2(x+3)$

$\frac{1}{3}x + 4 = 2x+6$

$-2 = \frac{5}{3}x$

$\boxed{-\frac{6}{5} = x}$

b) $3x+7=2(x+5)-(3-x)$

$3x+7 = 2x+10 \quad -3+x$

$3x+7 = 3x+7$

$0=0$

$\boxed{x = \mathbb{R}}$

33. Solve the formula for the indicated variable. Show your work.

$5x-3xy=8$ For x

$x(5-3y) = 8$

$x = \frac{8}{5-3y}$

34. Solve

a) $|2x+3|<2$

$2x+3<2$ and $2x+3>-2$

$2x < -1 \qquad 2x > -5$

$x < -1/2 \qquad x > -5/2$

$-5/2 < x < -1/2$

$\boxed{(-5/2, -1/2)}$

b) $|12x+4|\geq 8$

$12x+4 \geq 8$ or $12x+4 \leq -8$

$12x \geq 4 \qquad 12x \leq -12$

$x \geq 4/12 \qquad x \leq -1$

$x \geq 1/3$

$x \leq -1$ or $x \geq 1/3$

$\boxed{(-\infty, -1] \cup [1/3, \infty)}$