1. Simplify. Assume that no variable equals 0.

a. $(3x^2y^{-3})(-2x^3y^5)$

$= -6x^5y^2$

d. $\left(\frac{p^2r^3}{pr^4}\right)^2$

$= \left(\frac{p}{r}\right)^2$

$= \frac{p^2}{r^2}$

b. $\frac{3a^4b^3c}{6a^2b^5c^2}$

$= \frac{a^2}{2b^2c}$

e. $(4m^2 - 6m + 5) - (6m^2 + 3m - 1)$

$= -2m^2 - 9m + 6$

c. $4t(3rt - r)$

$= 12rt^2 - 4rt$

f. $(x + y)(x^2 + 2xy - y^2)$

$= x^3 + 2x^2y - xy^2 + x^2y + 2xy^2 - y^3$

$= x^3 + 3x^2y + xy^2 - y^3$

2. Find $3f(a-4) - 2h(a)$ if $f(x) = x^2 + 3x$ and $h(x) = 2x^2 - 3x + 5$

$= 3\left((a-4)^2 + 3(a-4)\right) - 2\left(2a^2 - 3a + 5\right)$

$= 3(a^2 - 8a + 16 + 3a - 12) - 4a^2 + 6a - 10$

$= 3(a^2 - 5a + 4) - 4a^2 + 6a - 10$

$= 3a^2 - 15a + 12 - 4a^2 + 6a - 10 = -a^2 - 9a + 2$

3. Simplify:

a. $(4r^3 - 8r^2 - 13r + 20) \div (2r - 5)$

$$\begin{array}{r|l} & 2r^2 + r - 4 \\ 2r-5 & 4r^3 - 8r^2 - 13r + 20 \\ & -(4r^3 - 10r^2) \\ & \quad 2r^2 - 13r \\ & \quad -(2r^2 - 5r) \\ & \qquad -8r + 20 \\ & \qquad -(-8r + 20) \\ & \qquad\qquad 0 \end{array}$$

$2r^2 + r - 4$

Is $(2r - 5)$ a factor? yes

b. $\frac{3x^3 - 16x^2 + 9x - 24}{x - 5}$

$$\begin{array}{r|rrrr} 5 & 3 & -16 & 9 & -24 \\ & & 15 & -5 & 20 \\ \hline & 3 & -1 & 4 & -4 \end{array}$$

$3x^2 - x + 4 + \frac{-4}{x-5}$

Is $(x - 5)$ a factor? No

c. $(6y^3 + 13y^2 - 10y - 24) \div (y + 2)$

$$\begin{array}{r|rrrr} -2 & 6 & 13 & -10 & -24 \\ & & -12 & -2 & 24 \\ \hline & 6 & 1 & -12 & 0 \end{array}$$

$6y^2 + y - 12$

d. $(a^4 + 5a^3 + 2a^2 - 6a + 4)(a + 2)^{-1}$

-2	1	5	2	-6	4
		-2	-6	8	-4
	1	3	-4	2	0

$a^3 + 3a^2 - 4a + 2$

e. $(4x^6 - 5x^4 + 3x^2 - x) \div (2\cancel{a}^{x} + 1)$

$$
\begin{array}{l}
\qquad\qquad 2x^5 - x^4 - 2x^3 + x^2 + x - 1 \\
2x + 1 \;\big)\; 4x^6 + 0x^5 - 5x^4 + 0x^3 + 3x^2 - x + 0 \\
\quad -(4x^6 + 2x^5) \\
\qquad -2x^5 - 5x^4 \\
\quad -(-2x^5 - x^4) \\
\qquad -4x^4 + 0x^3 \\
\quad -(-4x^4 - 2x^3) \\
\qquad 2x^3 + 3x^2 \\
\quad -(2x^3 + x^2) \\
\qquad 2x^2 - x \\
\quad -(2x^2 + x) \\
\qquad -2x + 0 \\
\quad -(-2x - 1) \\
\qquad 1
\end{array}
$$

$2x^5 - x^4 - 2x^3 + x^2 + x - 1 + \frac{1}{2x+1}$

4. Find $p(-3)$ if $p(x) = 4p^4 + 10p^3 + p - 5$

-3	4	10	0	1	-5
		-12	6	-18	51
	4	-2	6	-17	46

$\boxed{p(-3) = 46}$

5. Factor completely. If the polynomial is not factorable, write *prime.*

a. $a^4 - 16$

$(a^2 - 4)(a^2 + 4)$

$\boxed{(a-2)(a+2)(a^2+4)}$

b. $x^3 + 8y^3$

$\boxed{(x + 2y)(x^2 - 2yx + 4y^2)}$

c. $54x^3y - 16y^4$

$2y(27x^3 - 8y^3)$

$\boxed{2y(3x - 2y)(9x^2 + 6xy + 4y^2)}$

f. $(3k^4 + 27k^3) + (-7k - 63)$

$3k^3(k+9) - 7(k+9)$

$\boxed{(3k^3 - 7)(k+9)}$

d. $(6ay + 4by - 2cy) + (3az + 2bz - cz)$

$2y(3a + 2b - c) + z(3a + 2b - c)$

$\boxed{(2y + z)(3a + 2b - c)}$

g. $2x^4 - 16x$

$2x(x^3 - 8)$

$\boxed{2x(x-2)(x^2 + 2x + 4)}$

e. $2a^3 + 432$

$2(a^3 + 216)$

$\boxed{2(a+6)(a^2 - 6a + 36)}$

h. $6x^4 + 13x^2 - 5$

2, 3
6, 1
5, 1

$\boxed{(2x^2 + 5)(3x^2 - 1)}$

6. Solve each equation.

a. $x^3 + 2x^2 - 35x = 0$

$x(x^2 + 2x - 35) = 0$

$x(x+7)(x-5) = 0$

$\boxed{x = -7, 0, 5}$

c. $y^4 - 14y^2 + 45 = 0$

$(y^2 - 9)(y^2 - 5) = 0$

$(y-3)(y+3)(y^2 - 5) = 0$

$\boxed{y = \pm 3, \pm\sqrt{5}}$

b. $8x^4 - 10x^2 + 3 = 0$

$(4x^2 - 3)(2x^2 - 1) = 0$

$x = \pm\sqrt{3/4}, \pm\sqrt{1/2}$

$\boxed{x = \pm \frac{\sqrt{3}}{2}, \pm \frac{\sqrt{2}}{2}}$

d. $2x^3 - 12x^2 = -17x$

$2x^3 - 12x^2 + 17x = 0$

$x(2x^2 - 12x + 17) = 0$

$\cancel{x(2x \quad)(x \quad)} = 0$

$\boxed{x = 0}$ $x = \frac{12 \pm \sqrt{144 - 4(2)(17)}}{2(2)}$

$= \frac{12 \pm \sqrt{8}}{4}$

$= \frac{6 \pm \sqrt{2}}{2}$

$2x^4 - 16x = 0$

$2x(x^3 - 8) = 0$

$2x(x-2)(x^2+2x+4) = 0$

$x = 0 \quad x = 2 \qquad x = \dfrac{-2 \pm \sqrt{4 - 4(1)(4)}}{2}$

$= \dfrac{-2 \pm \sqrt{-12}}{2}$

$= \dfrac{-2 \pm 2\sqrt{3}\, i}{2}$

$= -1 \pm \sqrt{3}\, i$

$3k^4 + 27k^3 - 7k - 63 = 0$

$3k^3(k+9) - 7(k+9) = 0$

$(k+9)(3k^3 - 7) = 0$

$k = -9, \sqrt[3]{7/3}$