1) Let $f(x) = -x^2 + 4x + 1$, and $g(x) = \sqrt{x+2}$ find the following:

a) Domain of $f(x)$

$(-\infty, \infty)$

b) Domain of $g(x)$

$[-2, \infty)$

c) $f(0)$

$f(0) = 1$

d) $g(16) = \sqrt{16+2}$

$= \sqrt{18}$

e) $f(-1) = -(-1)^2 + 4(-1) + 1$

$= -1 - 4 + 1$

$= -4$

f) $g(23) = \sqrt{23+2}$

$= 5$

g) $f(g(2))$

$= f(\sqrt{4})$

$= -2^2 + 4(2) + 1$

$= 5$

h) $g(f(2)) = g(-(2)^2 + 4(2) + 1)$

$= g(5)$

$= \sqrt{7}$

i) $f(g(x))$

$= -(\sqrt{x+2})^2 + 4\sqrt{x+2} + 1$

$= -x - 2 + 4\sqrt{x+2} + 1$

$= 4\sqrt{x+2} - x - 1$

j) $g(f(x))$

$= \sqrt{(-x^2 + 4x + 1) + 2}$

$= \sqrt{-x^2 + 4x + 3}$

k) $f(f(-3))$

$= f(-(-3)^2 + 4(-3) + 1)$

$= f(-9 - 12 + 1)$

$= f(-20)$

$= -(-20)^2 + 4(-20) + 1$

$= -479$

l) $g(g(81)) = g(\sqrt{81+2})$

$= g(\sqrt{83})$

$= \sqrt{\sqrt{83} + 2}$

m) $f(a+h)$

$= -(a+h)^2 + 4(a+h) + 1$

$= -a^2 - 2ah - h^2 + 4a + 4h + 1$

n) $\frac{f(a+h) - f(a)}{h}$

$= \frac{(-a^2 - 2ah - h^2 + 4a + 4h + 1) - (-a^2 + 4a + 1)}{h}$

$= \frac{-2ah - h^2 + 4h}{h}$

$= -2a - h + 4$

2) **Temperature Conversion** The formula to convert temperatures from degrees Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$. Write the inverse function, which converts temperatures from Fahrenheit to Celsius. What is the Celsius temperature that is equal to 94 degrees Fahrenheit?

$C = \frac{9}{5} F + 32$

$C - 32 = \frac{9}{5} F$

$\frac{5}{9}(C - 32) = F$

$\frac{5}{9} C - \frac{160}{9} = F^{-1}$

$\frac{5}{9}(94) - \frac{160}{9} = F^{-1}$

$\boxed{34.4°\ C\ =\ 94°\ F}$

3) Find the inverse of the function. Verify your answer.

a) $f(x) = 3 - 2x$

$x = 3 - 2y$

$x - 3 = -2y$

$\frac{3}{2} - \frac{1}{2}x = y$

$\frac{3-x}{2} = y^{-1}$

verify:

$f(f^{-1}(x)) = 3 - 2\left(\frac{3-x}{2}\right)$

$= 3 - 3 + x$

$= x\ \checkmark$

$f^{-1}(f(x)) = \frac{3-(3-2x)}{2}$

$= \frac{3 - 3 + 2x}{2}$

$= x\ \checkmark$

b) $g(x) = \frac{4-x}{3x}$

$x = \frac{4-y}{3y}$

$3yx = 4 - y$

$3yx + y = 4$

$y(3x + 1) = 4$

$g^{-1} = \frac{4}{3x+1}$

*won't need to verify this complex of inverse functions

c) $h(x) = \sqrt[5]{5x + 4}$

$x = \sqrt[5]{5y + 4}$

$x^5 = 5y + 4$

$\frac{x^5 - 4}{5} = h^{-1}(x)$

verify:

$h(h^{-1}(x)) = \sqrt[5]{5\left(\frac{x^5 - 4}{5}\right) + 4}$

$= \sqrt[5]{x^5 - 4 + 4}$

$= \sqrt[5]{x^5}$

$= x\ \checkmark$

$h^{-1}(h(x)) = \frac{\sqrt[5]{5x+4}^5 - 4}{5}$

$= \frac{5x + 4 - 4}{5}$

$= \frac{5x}{5}$

$= x\ \checkmark$

4) On a certain day, the function that gives Japanese yen in terms of U.S. dollars is $y = 110.16d$ where y represents yen and d represents dollars.

a) Find the inverse function.

$$d = 110.16y$$
$$\frac{1}{110.16} d = y^{-1}$$

$$\frac{1}{110.16} y = d$$

b) How many dollars do you receive for 412 yen?

$$\frac{1}{110.16} (412) = \$3.74$$

c) The function that gives Indian rupees in terms of U.S. dollars if $r = 43.94d$ where r represents rupees and d represents dollars. How many rupees do you receive for 529 yen?

$$\frac{1}{110.16} (529) = \$4.8021$$

$$r = 43.94 (4.8021)$$
$$\boxed{r = 211.00}$$

5) Use the table below to answer each of the following questions.

x	0	1	2	3
$f(x)$	2	3	5	10
$g(x)$	5	3	1	0

a) $g(f(0)) \rightarrow g(2)$

$$= 1$$

b) $f(g(2)) = f(1)$

$$\rightarrow 3$$

*remember x and y swap for inverses

c) $f^{-1}(2) = 0$

$f: (0, 2)$

$\Rightarrow f^{-1}: (2, 0)$

d) $g^{-1}(0) = 3$

$g: (3, 0)$

$g^{-1}: (0, 3)$

6) Let $f(x) = 2x^2 + 3x + 1$ Find

a) $f(a) = 2a^2 + 3a + 1$

b) $f(a+h) = 2(a+h)^2 + 3(a+h) + 1$

$= 2a^2 + 4ah + 2h^2 + 3a + 3h + 1$

c) $f(a+h) - f(a)$

$= 2a^2 + 4ah + 2h^2 + 3a + 3h + 1$
$- (2a^2 + 3a + 1)$

$= 4ah + 2h^2 + 3h$

d) $\frac{f(a+h) - f(a)}{h}$

$= \frac{4ah + 2h^2 + 3h}{h}$

$= 4a + 2h + 3$

7) Write the expression in the simplest form:

a) $\sqrt{36x^5}$
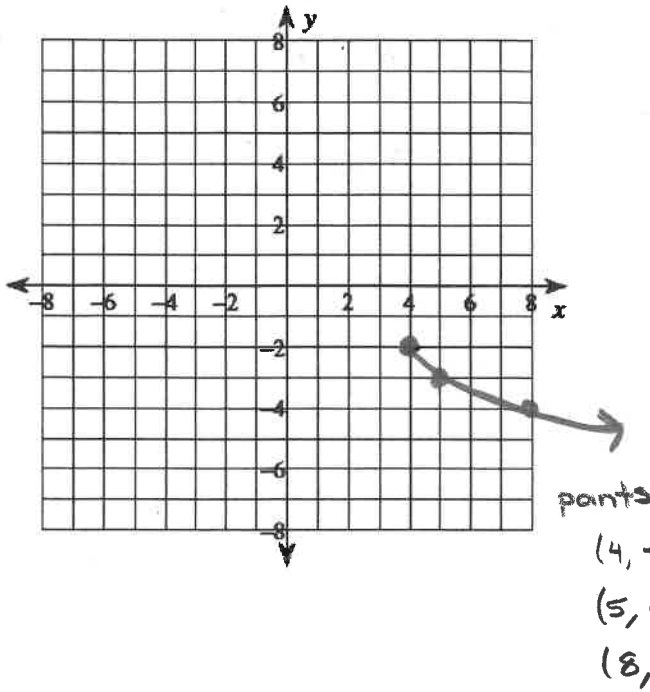
$= 6x^2 \sqrt{x}$

b) $\sqrt[3]{x^4 y^6 z}$

$= xy^2 \sqrt[3]{xz}$

8) Simplify. Assume all variable are positive.

a) $\sqrt[3]{81x^7}$

$= 3x^2 \sqrt[3]{x}$

b) $\sqrt[5]{64x^7 y^{15} z^{21}} = 2xy^3 z^4 \sqrt[5]{2x^2 z}$

harder than on
test see ? #18
for problems that
reflect test ques.

9) Graph the function $y = -\sqrt{x-4}-2$. Find the domain and range. Describe the transformations from the parent function $y = \sqrt{x}$
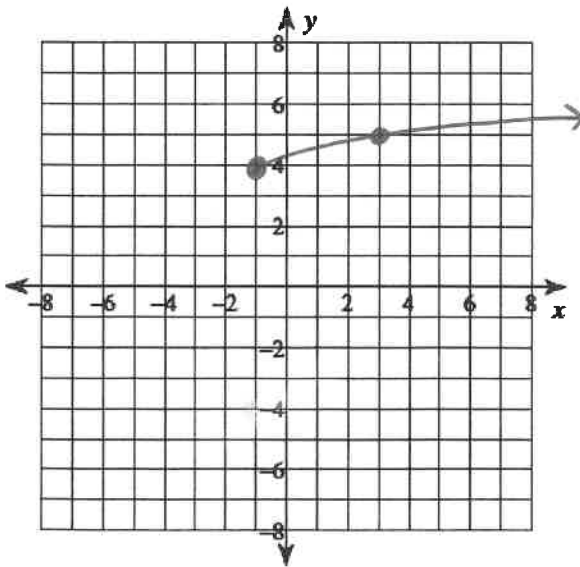

Domain: __[4, ∞)__

Range: __(-∞, -2]__

Transformations: __right 4__
down 2
reflected over x-axis

points
(4, -2)
(5, -3)
(8, -4)

13) Graph the function $y = \frac{1}{2}\sqrt{x+1} + 4$. Find the domain and range. Describe the transformations from the parent function $y = \sqrt{x}$
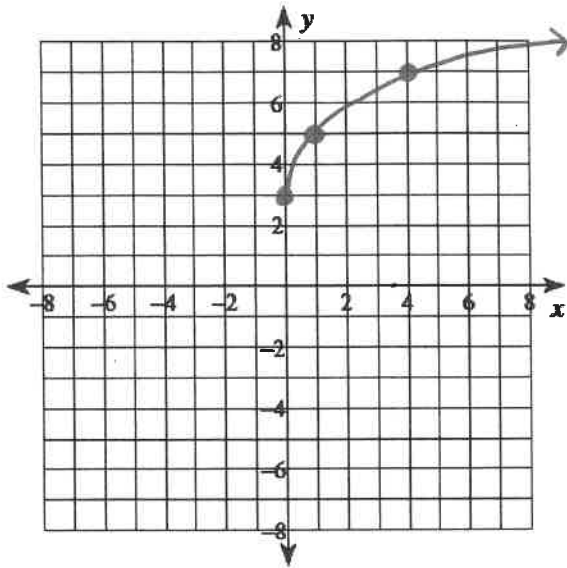

Domain: __[-1, ∞)__

Range: __[4, ∞)__

Transformations: __left 1__
up 4
vertically shrunk
by ½

points
(-1, 4)
(3, 5)

14) Graph the function $y = 2\sqrt{x} + 3$. Find the domain and range. Describe the transformations from the parent function $y = \sqrt{x}$

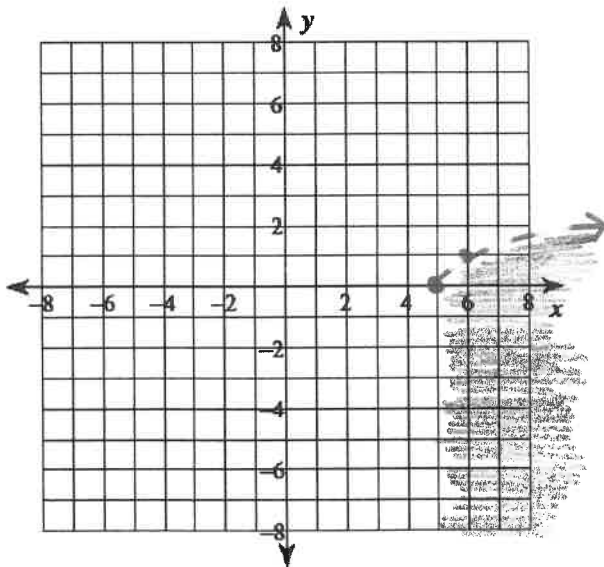Domain: __[0, ∞)__

Range: __[3, ∞)__

Transformations: __up 3__
vertically stretched
by 2

points
(0, 3)
(1, 5)
(4, 7)

15) Graph the function $y < \sqrt{x - 5}$. Find the domain and range. Describe the transformations from the parent function $y = \sqrt{x}$

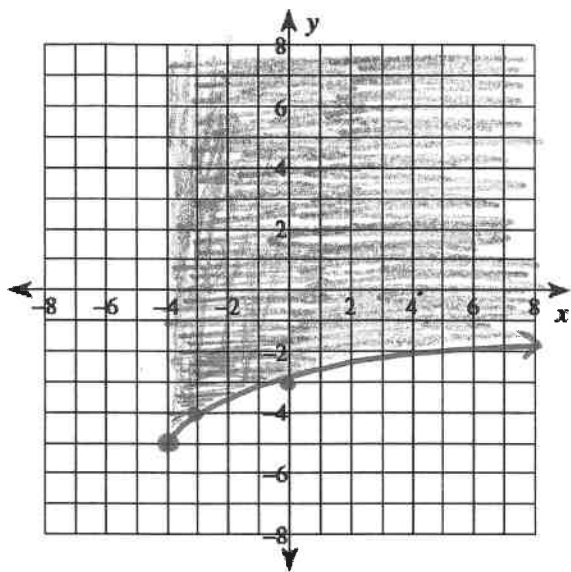Domain: __[5, ∞)__

Range: __[0, ∞)__

Transformations: __right 5__

16) Graph the function $y \geq \sqrt{x+4} - 5$. Find the domain and range. Describe the transformations from the parent function $y = \sqrt{x}$
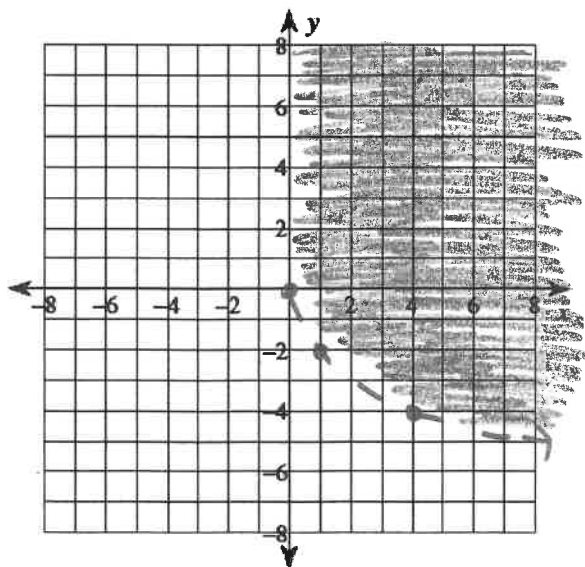

Domain: __[-4, ∞)__

Range: __[-5, ∞)__

Transformations: __left 4__
                     down 5

17) Graph the function $y > -2\sqrt{x}$. Find the domain and range. Describe the transformations from the parent function $y = \sqrt{x}$

Domain: __[0, ∞)__

Range: __[0, ∞)__

Transformations: __reflect over x-axis__
                   vertical stretch of 2

18) Simplify the following:

a) $\pm\sqrt{121a^4b^{18}}$

$$= \pm\, 11\, a^2 b^9$$

b) $\sqrt{(x^4+3)^{12}}$

$$= \left| (x^4+3)^6 \right|$$

$$= \left(x^4 + 3 \right)^6$$

c) $\sqrt[3]{27(2x-5)^{15}}$

$$= 3(2x-5)^5$$

d) $\sqrt[5]{-(y-6)^{20}}$

$$= -(y-6)^4$$

e) $\sqrt[3]{8(x+4)^6}$

$$= 2(x+4)^2$$

f) $\sqrt[4]{16(y+x)^8}$

$$= \left| 2(y+x)^2 \right|$$

$$= 2(y+x)^2$$