1. Use $\log_5 2 \approx 0.4307$ to approximate the value of $\log_5 4$

$$\log_5(4) = \log_5(2 \cdot 2) = \log_5 2 + \log_5 2$$
$$= \boxed{0.8614}$$

2. Use $\log_5 2 \approx 0.4307$ and $\log_5 3 \approx 0.6826$ to approximate the value of:

a. $\log_5 \frac{15}{4}$

15
$\wedge$
5 3

$$= \log_5 \frac{(5 \cdot 3)}{4}$$
$$= \log_5 5 + \log_5 3 - \log_5 2^2$$
$$= 1 + 0.6826 - 2(0.4307) = 0.8212$$

b. $\log_5 6 = \log_5 2 \cdot 3$
$$= \log_5 2 + \log_5 3$$
$$= \boxed{1.1133}$$

c. $\log_5 72$

72
$\wedge$
9 8

$$= \log_5 3^2 \cdot 2^3$$
$$= 2\log_5 3 + 3\log_5 2$$
$$= 2(0.6826) + 3(0.4307)$$
$$= \boxed{2.6573}$$

3. Expand the following expressions

a. $\ln(x^6 y^3)$
$$= \ln x^6 + \ln y^3$$
$$= \boxed{6\ln x + 3\ln y}$$

b. $\log_7\left(\frac{x^3}{y}\right)^3$
$$= 3 \log_7 \frac{x^3}{y}$$
$$= 3\log_7 x^3 - 3\log_7 y = \boxed{9 \log_7 x - 3\log_7 y}$$

c. $\log_6(c^5 \cdot \sqrt[3]{a})$
$$= \log_6 c^5 + \log_6 a^{1/3}$$
$$= \boxed{5\log_6 c + \frac{1}{3} \log_6 a}$$

4. Condense the following expressions

a. $4\log_2 11 - 6\log_2 6$
$$= \log_2 11^4 - \log_2 6^6$$
$$= \boxed{\log_2 \frac{11^4}{6^6}}$$

b. $\log_7 z + \frac{1}{3}\log_7 x - \log_7 2$
$$= \log_7 z + \log_7 x^{1/3} - \log_7 2$$
$$= \log_7 z x^{1/3} - \log_7 2$$
$$= \boxed{\log_7 \frac{z x^{1/3}}{2}}$$

5. Solve: *always check answers!*

(left margin, vertical text) * Must show work to get full credit b/c calculator active test

a. $\log_6 10 + \log_6 x = \log_6 40$

$\log_6 10x = \log_6 40$

$10x = 40$

$\boxed{x = 4}$

b. $4^x = 20$

$\log_4 4^x = \log_4 20$

$x = \log_4 20$

$\boxed{x \approx 2.1609}$

c. $\ln 3x = 1$

$e^1 = 3x$

$\boxed{\frac{1}{3}e = x}$

$x \approx 0.90609$

d.
$\log_4(m - 3) + \log_4(m + 3) = 2$

$\log_4 (m-3)(m+3) = 2$

$4^2 = (m-3)(m+3)$

$16 = m^2 - 9$

$25 = m^2$
$\pm 5 = m$ $\boxed{m = 5}$

e. $6^{3n} = 43^{5n-4}$

$\log_{43} 6^{3n} = \log_{43} 43^{5n-4}$

$3n \log_{43} 6 = 5n - 4$

$3n \log_{43} 6 - 5n = -4$

$n(3\log_{43} 6 - 5) = -4$

$n = \dfrac{-4}{3\log_{43} 6 - 5} \approx \boxed{1.12017}$

f. $4 + 3e^{5x} = 27$

$3e^{5x} = 23$

$e^{5x} = 23/3$

$\ln(e^{5x}) = \ln(23/3)$

$5x = \ln 23/3$

$x = \frac{1}{5}\ln 23/3$

$\boxed{x \approx 0.40737}$

g. $\log_5 n = \frac{1}{3}\log_5 64 + \frac{1}{2}\log_5 49$

$\log_5 n = \log_5 \sqrt[3]{64} + \log_5 \sqrt{49}$

$\log_5 n = \log_5 4 \cdot 7$

$\boxed{n = 28}$

j. $7^{n+3} = 80$

$\log_7 7^{n+3} = \log_7 80$

$n + 3 = \log_7 80$

$n = \log_7 80 - 3$

$\boxed{n \approx -0.748}$

h. $\log_6(5 - 2a) - \log_6(3a) = 1$

$\log_6 \frac{5-2a}{3a} = 1$

$6^1 = \frac{5-2a}{3a}$

$18a = 5 - 2a$

$20a = 5$

$\boxed{a = \frac{1}{4}}$

k. $3^n = 6^{n-2}$

$\log_6 3^n = \log_6 6^{n-2}$

$n \log_6 3 = (n-2)$

$n \log_6 3 - n = -2$

$n(\log_6 3 - 1) = -2$

$n = \frac{-2}{\log_6 3 - 1} \approx \boxed{5.1699}$

i. $\log_3(x - 3) + \log_3(x + 2) = \log_3 6$

$\log_3 (x-3)(x+2) = \log_3 6$

$x^2 - x - 6 = 6$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x = -3, 4$

$\boxed{x = 4}$

l. $\ln(x + 4) = 4$

$e^4 = x + 4$

$e^4 - 4 = x$

$\boxed{50.598 \approx x}$

$$\left(\tfrac{1}{3}\right)^{x} = 9$$

6. Evaluate:

    a. $3\log_2 64 + e^{\ln 5} + \log_{1/3} 9$

$$3(5) \; + \; 5 \; + \; (-2)$$

$$15 \; + 5 \; - \; 2$$

$$\boxed{18}$$

7. Solve:

    a. $5^{\log_5 2x - \log_5 (x-3)} = \ln e^{x+4}$

$$5^{\log_5 \frac{2x}{x-3}}$$

$$= x+4$$

$$\frac{2x}{x-3} = x+4$$

$$2x = x^2 + x - 12$$

$$0 = x^2 - x - 12$$

$$0 = (x-4)(x+3)$$

$$x = 4, \; -3$$

$$\boxed{x = 4}$$

Challenge:

Solve $\log_x \left[\log_2 (\log_3 81)\right] = 2$

$$\log_x \left[\log_2 (4)\right] = 2$$

$$\log_x (2) = 2$$

$$x^2 = 2$$

$$\boxed{x = \sqrt{2}}$$