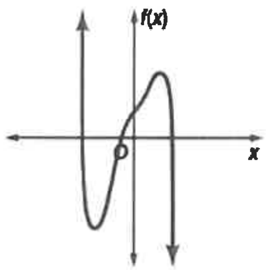
1. Describe the end behavior of the graph. Then determine whether it represents an odd-degree or an even-degree polynomial function and state the number of real zeros.



End Behavior:

$x \to \infty$, $f(x) \to$ $-\infty$

$x \to -\infty$, $f(x) \to$ ∞

Degree: odd

Number of real zeros: 3

2. Find $3f(a-4) - 2h(a)$ if $f(x) = x^2 + 3x$ and $h(x) = 2x^2 - 3x + 5$

$$3[(a-4)^2 + 3(a-4)] - 2[2(a)^2 - 3(a) + 5]$$

$$= 3(a^2 - 8a + 16 + 3a - 12) - 2(2a^2 - 3a + 5)$$

$$= 3(a^2 - 5a + 4) - 4a^2 + 6a - 10$$

$$= 3a^2 - 15a + 12 - 4a^2 + 6a - 10$$

$$= \boxed{-a^2 - 9a + 2}$$

3. Describe each of the following end behaviors in words and with a sketch.

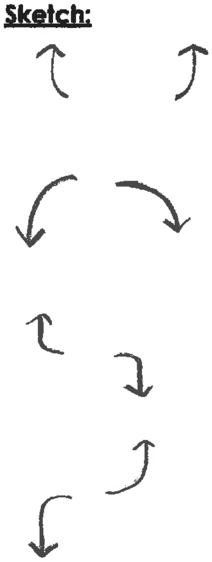
Word Description: **Sketch:**

1. Positive Even Function: $x \to \infty$ $f(x) \to \infty$
$x \to -\infty$ $f(x) \to \infty$

2. Positive Odd Function: $x \to \infty$ $f(x) \to -\infty$
$x \to -\infty$ $f(x) \to -\infty$

3. Negative ~~Even~~ Odd Function: $x \to \infty$ $f(x) \to -\infty$
$x \to -\infty$ $f(x) \to \infty$

4. Negative ~~Odd~~ Even Function: $x \to \infty$ $f(x) \to \infty$
$x \to -\infty$ $f(x) \to -\infty$



4. Write in standard form: $f(x) = 3x - 5 + x^3 - 2x^2$. What is the degree and leading coefficient of the polynomial? What is the end behavior of the polynomial?

Standard form: $f(x) = \cancel{x}x^3 - 2x^2 + 3x - 5$

Degree: 3

Leading Coefficient: 1

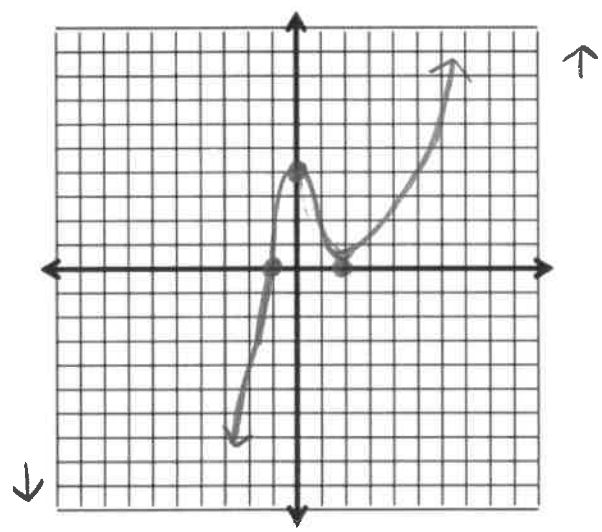
End Behavior:

$x \to \infty$, $f(x) \to$ ∞

$x \to -\infty$, $f(x) \to$ $-\infty$

5. Graph the following functions. After the function is graphed, determine the domain and range. Label all zeros and y-intercepts clearly.

a. $f(x) = (x + 1)(x - 2)^2$



Degree: 3

Sign of the Leading Coefficient: pos

Y-intercept: (0, 4)

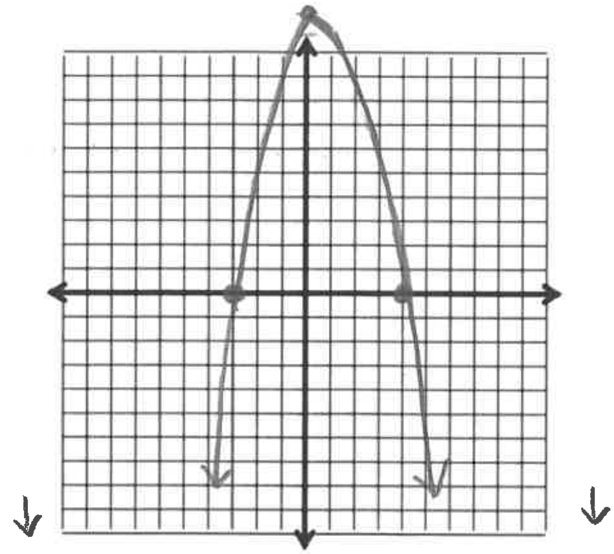
Zeros: (-1, 0) (2, 0)

Multiplicity of zeros: 1, 2

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

b. $f(x) = -(x-4)(x+3)$



Degree: 2

Sign of the Leading Coefficient: neg

Y-intercept: (0, 12)

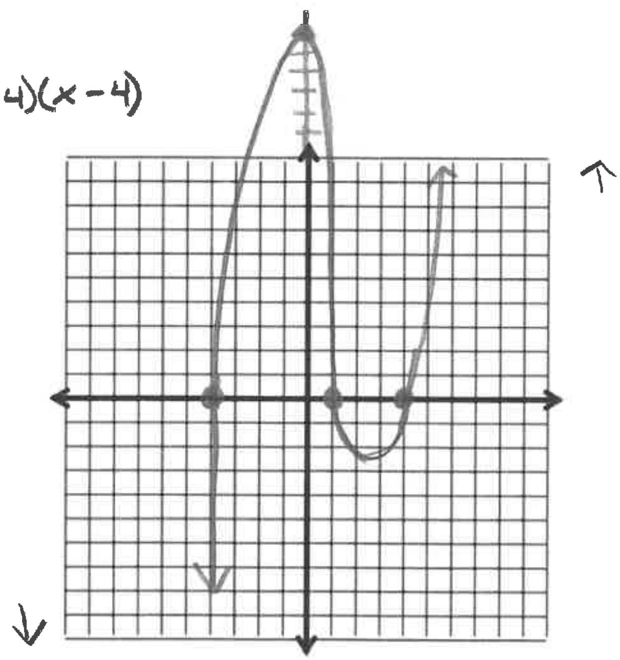
Zeros: (4, 0)   (-3, 0)

Multiplicity of zeros: both 1

Domain: $(-\infty, \infty)$

Range: $(-\infty, 12]$

c. $f(x) = (x-1)(x^2-16) = (x-1)(x+4)(x-4)$



Degree: 3

Sign of the Leading Coefficient: pos

Y-intercept: (0, 16)

Zeros: (1, 0) (4, 0)   (-4, 0)

Multiplicity of zeros: all 1

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

6. Determine the zeros, maxima, minima, smallest possible degree, domain, and range of the function below:

Zeros: (-3, 0) (-1, 0) (1, 0) (3, 0)

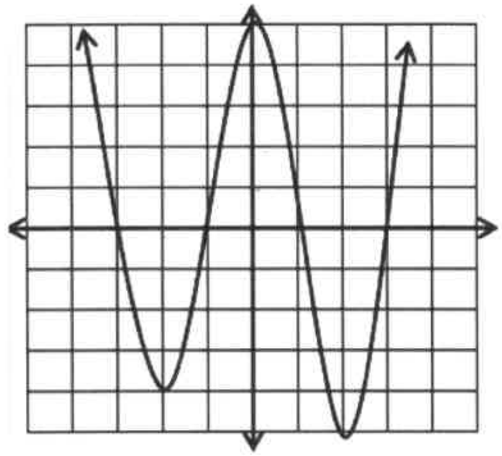
Maxima: (0, 5)

Minima: (-2, -4) (2, -5)

Smallest Possible Degree: 4

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$



7. Factor $3x^3 - 4x^2 - 28x - 16$ completely given $x + 2$ is a factor.

-2	3	-4	-28	-16
		-6	20	16
	3	-10	-8	0

$f(x) = (x+2)(3x^2 - 10x - 8)$

$= \boxed{(x+2)(3x+2)(x-4)}$

8. Factor $3x^3 - 16x^2 + 3x + 10$ completely given $x = 5$ is a zero.

5	3	-16	3	10
		15	-5	-10
	3	-1	-2	0

$f(x) = (x-5)(3x^2 - x - 2)$

$= \boxed{(x-5)(3x+2)(x-1)}$

9. Find all of the zeros of the function $f(x) = x^3 - 2x^2 - 23x + 60$, given $f(3) = 0$.

(3, 0)
x = 3 is a zero

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -23 & 60 \\ & & 3 & 3 & -60 \\ \hline & 1 & 1 & -20 & 0 \end{array}$$

$f(x) = (x-3)(x^2+x-20)$

$= (x-3)(x+5)(x-4)$

$x = 3, -5, 4$

10. Find the other zeros of the function given that one of the zeros is 7.
$f(x) = 10x^3 - 81x^2 + 71x + 42.$

$$\begin{array}{r|rrrr} 7 & 10 & -81 & 71 & 42 \\ & & 70 & -77 & -42 \\ \hline & 10 & -11 & -6 & 0 \end{array}$$

$f(x) = (x-7)(10x^2 - 11x - 6)$

$= (x-7)(2x-3)(5x+2)$

$x = 7, 3/2, -2/5$

11. Find all of the possible rational zeros of the function $f(x) = 5x^4 - 9x^3 + 3x^2 - 6x + 20$

$$\frac{\text{factors of } 20}{\text{factors of } 5} = \frac{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20}{\pm 1, \pm 5}$$

$= \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 1/5, \pm 2/5, \pm 4/5$

12. Find all of the zeros of the functions below:

a. $f(x) = x^3 - 4x^2 + x + 6$ $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{c|cccc} 2 & 1 & -4 & 1 & 6 \\ & & 2 & -4 & -6 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$f(x) = (x-2)(x^2-2x-3)$

$= (x-2)(x-3)(x+1)$

$\boxed{x = -1, 2, 3}$

b. $f(x) = x^3 + 2x^2 + 4x + 8$ $\pm 1, \pm 2, \pm 4, \pm 8$

$$\begin{array}{c|cccc} 2 & 1 & 2 & 4 & 8 \\ & & 2 & 8 & \\ \hline & 1 & 4 & & \end{array}$$

$$\begin{array}{c|cccc} -2 & 1 & 2 & 4 & 8 \\ & & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$f(x) = (x+2)(x^2+4)$

$\boxed{x = -2, \pm 2i}$

13. Write a polynomial function with a leading coefficient of 1 and the following zeros. Write in factored form and standard form.

a. 1, − 2, 6

Factored Form: $f(x) = (x-1)(x+2)(x-6)$

$f(x) = (x-1)(x+2)(x-6)$

$= (x^2+x-2)(x-6)$

$= x^3 + x^2 - 2x - 6x^2 - 6x + 12$

$= x^3 - 5x^2 - 8x + 12$

Standard Form: $f(x) = x^3 - 5x^2 - 8x + 12$

b. 0, − 2, 3*i*, $-3i$

$f(x) = x(x+2)(x-3i)(x+3i)$

Factored Form: $f(x) = x(x+2)(x-3i)(x+3i)$

$= (x^2+2x)(x-3i)(x+3i)$

$= (x^2+2x)(x^2-9i^2)$

$= (x^2+2x)(x^2+9)$

$= x^4 + 2x^3 + 9x^2 + 18x$

Standard Form: $f(x) = x^4 + 2x^3 + 9x^2 + 18x$

c. $1, 2-i$, $2+i$

Factored Form: $f(x) = (x-1)(x-(2-i))(x-(2+i))$

$f(x) = (x-1)(x-2+i)(x-2-i)$

$= (x-1)(x^2-4x+4-i^2)$

$= (x-1)(x^2-4x+5)$

$= x^3-4x^2+5x-x^2+4x-5$

$= x^3-5x^2+9x-5$

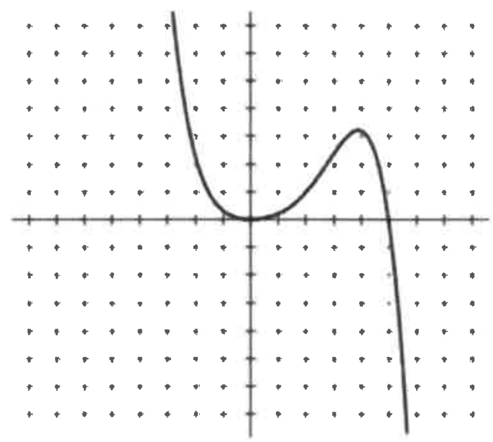
	x	-2	i
x	x^2	$-2x$	ix
-2	$-2x$	4	$-2i$
$-i$	$-ix$	$2i$	$-i^2$

Standard Form: $f(x) = x^3-5x^2+9x-5$

14. Write a possible equation given the graph:

Zeros: $(0,0)$ $(5,0)$

Equation: $f(x) = -x^2(x-5)$



15. Determine how many possible positive, negative, and imaginary zeros the following functions have:

a. $f(x) = x^5 + 2x^4 - 3x^3 + x^2 + 10$

$f(x) = x^5 + 2x^4 - 3x^3 + x^2 + 10$

2

$f(-x) = -x^5 + 2x^4 + 3x^3 + x^2 + 10$

1

2 pos 1 neg 2 imaginary

0 pos 1 neg 4 imaginary

b. $f(x) = x^4 + x^3 - 2x^2 - 9x - 1$

1

$f(-x) = x^4 - x^3 - 2x^2 + 9x - 1$

3

1 pos 3 neg

1 pos 1 neg 2 imaginary