NAME _____ DATE _____ PERIOD _____

4-1 Skills Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

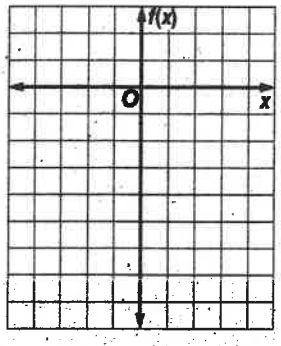
b. Make a table of values that includes the vertex.

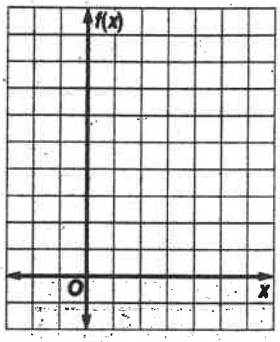
c. Use this information to graph the function.

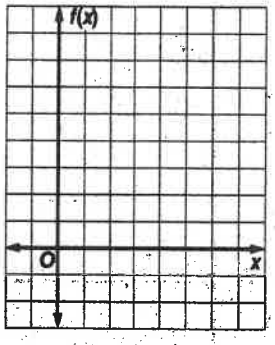
1. $f(x) = -2x^2$

2. $f(x) = x^2 - 4x + 4$

3. $f(x) = x^2 - 6x + 8$







Determine whether each function has a maximum or a minimum value, and find that value. Then state the domain and range of the function.

4. $f(x) = 6x^2$

5. $f(x) = -8x^2$

6. $f(x) = x^2 + 2x$

7. $f(x) = -2x^2 + 4x - 3$

8. $f(x) = 3x^2 + 12x + 3$

9. $f(x) = 2x^2 + 4x + 1$

10. $f(x) = 3x^2$

11. $f(x) = x^2 + 1$

12. $f(x) = -x^2 + 6x - 15$

13. $f(x) = 2x^2 - 11$

14. $f(x) = x^2 - 10x + 5$

15. $f(x) = -2x^2 + 8x + 7$

Lesson 4-1

NAME _____ DATE _____ PERIOD _____

4-1 Practice

Graphing Quadratic Functions

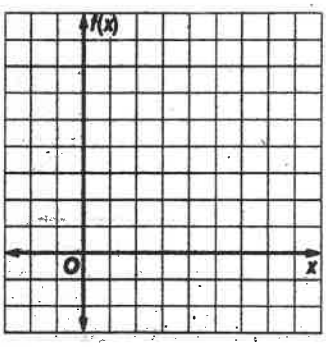
Complete parts a–c for each quadratic function.

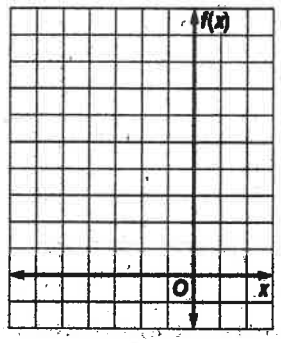
a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

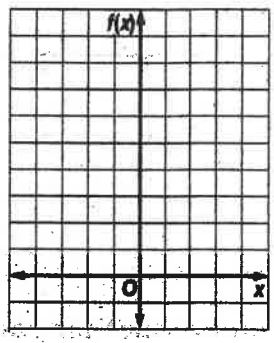
b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = x^2 - 8x + 15$ **2.** $f(x) = -x^2 - 4x + 12$ **3.** $f(x) = 2x^2 - 2x + 1$







Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

4. $f(x) = x^2 + 2x - 8$ **5.** $f(x) = x^2 - 6x + 14$ **6.** $v(x) = -x^2 + 14x - 57$

7. $f(x) = 2x^2 + 4x - 6$ **8.** $f(x) = -x^2 + 4x - 1$ **9.** $f(x) = -\frac{2}{3}x^2 + 8x - 24$

10. GRAVITATION From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height $h(t)$ of the ball t seconds after Susan throws it is given by $h(t) = -16t^2 + 32t + 4$. For $t \geq 0$, find the maximum height reached by the ball and the time that this height is reached.

11. HEALTH CLUBS Last year, the SportsTime Athletic Club charged \$20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each \$1 increase in the price.

a. What price should the club charge to maximize the income from the aerobics classes?

b. What is the maximum income the SportsTime Athletic Club can expect to make?

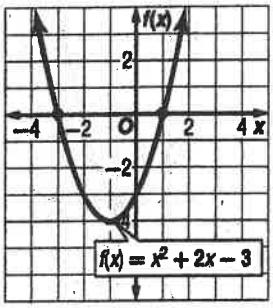
NAME _____ DATE _____ PERIOD _____

4-2 Skills Practice

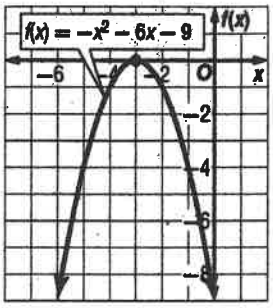
Solving Quadratic Equations By Graphing

Use the related graph of each equation to determine its solutions.

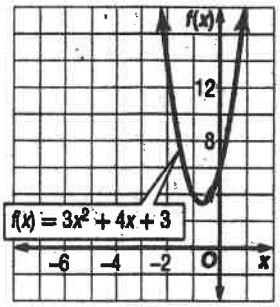
1. $x^2 + 2x - 3 = 0$



2. $-x^2 - 6x - 9 = 0$

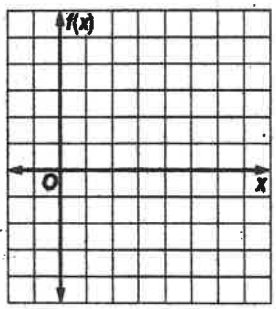


3. $3x^2 + 4x + 3 = 0$

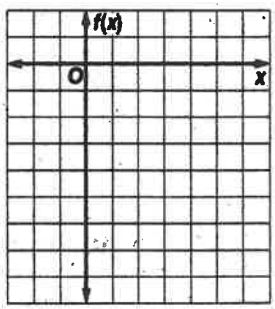


Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

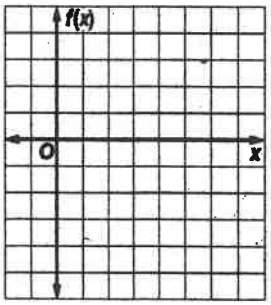
4. $x^2 - 6x + 5 = 0$



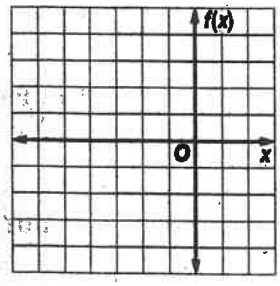
5. $-x^2 + 2x - 4 = 0$



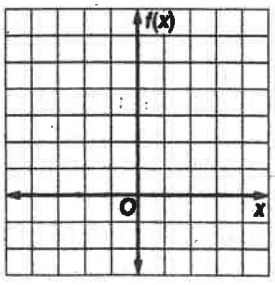
6. $x^2 - 6x + 4 = 0$



7. $-x^2 - 4x = 0$



8. $-x^2 + 36 = 0$



Lesson 4-2

NAME _____ DATE _____ PERIOD _____

4-3 Skills Practice

Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1. 1, 4

2. 6, −9

3. −2, −5

4. 0, 7

5. $-\frac{1}{3}$, −3

6. $-\frac{1}{2}, \frac{3}{4}$

Factor each polynomial.

7. $m^2 + 7m - 18$

8. $2x^2 - 3x - 5$

9. $4z^2 + 4z - 15$

10. $4p^2 + 4p - 24$

11. $3y^2 + 21y + 36$

12. $c^2 - 100$

Solve each equation by factoring.

13. $x^2 = 64$

14. $x^2 - 100 = 0$

15. $x^2 - 3x + 2 = 0$

16. $x^2 - 4x + 3 = 0$

17. $x^2 + 2x - 3 = 0$

18. $x^2 - 3x - 10 = 0$

19. $x^2 - 6x + 5 = 0$

20. $x^2 - 9x = 0$

21. $x^2 - 4x = 21$

22. $2x^2 + 5x - 3 = 0$

23. $4x^2 + 5x - 6 = 0$

24. $3x^2 - 13x - 10 = 0$

25. **NUMBER THEORY** Find two consecutive integers whose product is 272.

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4-4 Skills Practice

Complex Numbers

Simplify.

1. $\sqrt{99}$

2. $\sqrt{\frac{27}{49}}$

3. $\sqrt{52x^3y^5}$

4. $\sqrt{-108x^7}$

5. $\sqrt{-81x^6}$

6. $\sqrt{-23} \cdot \sqrt{-46}$

7. $(3i)(-2i)(5i)$

8. i^{11}

9. i^{65}

10. $(7 - 8i) + (-12 - 4i)$

11. $(-3 + 5i) + (18 - 7i)$

12. $(10 - 4i) - (7 + 3i)$

13. $(7 - 6i)(2 - 3i)$

14. $(3 + 4i)(3 - 4i)$

15. $\frac{8 - 6i}{3i}$

16. $\frac{3i}{4 + 2i}$

Solve each equation.

17. $3x^2 + 3 = 0$

18. $5x^2 + 125 = 0$

19. $4x^2 + 20 = 0$

20. $-x^2 - 16 = 0$

21. $x^2 + 18 = 0$

22. $8x^2 + 96 = 0$

Find the values of ℓ and m that make each equation true.

23. $20 - 12i = 5\ell + (4m)i$

24. $\ell - 16i = 3 - (2m)i$

25. $(4 + \ell) + (2m)i = 9 + 14i$

26. $(3 - m) + (7\ell - 14)i = 1 + 7i$

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4-4 Practice

Complex Numbers

Simplify.

1. $\sqrt{-36}$
2. $\sqrt{-8} \cdot \sqrt{-32}$
3. $\sqrt{-15} \cdot \sqrt{-25}$
4. $(-3i)(4i)(-5i)$
5. $(7i)^2(6i)$
6. i^{42}
7. i^{55}
8. i^{89}
9. $(5 - 2i) + (-13 - 8i)$
10. $(7 - 6i) + (9 + 11i)$
11. $(-12 + 48i) + (15 + 21i)$
12. $(10 + 15i) - (48 - 30i)$
13. $(28 - 4i) - (10 - 30i)$
14. $(6 - 4i)(6 + 4i)$
15. $(8 - 11i)(8 - 11i)$
16. $(4 + 3i)(2 - 5i)$
17. $(7 + 2i)(9 - 6i)$
18. $\frac{6 + 5i}{-2i}$
19. $\frac{2}{7 - 8i}$
20. $\frac{3 - i}{2 - i}$
21. $\frac{2 - 4i}{1 + 3i}$

Solve each equation.

22. $5n^2 + 35 = 0$
23. $2m^2 + 10 = 0$
24. $4m^2 + 76 = 0$
25. $-2m^2 - 6 = 0$
26. $-5m^2 - 65 = 0$
27. $\frac{3}{4}x^2 + 12 = 0$

Find the values of ℓ and m that make each equation true.

28. $15 - 28i = 3\ell + (4m)i$
29. $(6 - \ell) + (3m)i = -12 + 27i$
30. $(3\ell + 4) + (3 - m)i = 16 - 3i$
31. $(7 + m) + (4\ell - 10)i = 3 - 6i$

32. **ELECTRICITY** The impedance in one part of a series circuit is $1 + 3j$ ohms and the impedance in another part of the circuit is $7 - 5j$ ohms. Add these complex numbers to find the total impedance in the circuit.

33. **ELECTRICITY** Using the formula $E = IZ$, find the voltage E in a circuit when the current I is $3 - j$ amps and the impedance Z is $3 + 2j$ ohms.

Lesson 4-4

NAME _____ DATE _____ PERIOD _____

4-1 Study Guide and Intervention *(continued)*

Graphing Quadratic Functions

Maximum and Minimum Values The y-coordinate of the vertex of a quadratic function is the **maximum value** or **minimum value** of the function.

Maximum or Minimum Value of a Quadratic Function	The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$, opens up and has a minimum when $a > 0$. The graph opens down and has a maximum when $a < 0$.

Example **Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.**

a. $f(x) = 3x^2 - 6x + 7$

For this function, $a = 3$ and $b = -6$.

Since $a > 0$, the graph opens up, and the function has a minimum value.

The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = \frac{-(-6)}{2(3)} = 1$.

Evaluate the function at $x = 1$ to find the minimum value.

$f(1) = 3(1)^2 - 6(1) + 7 = 4$, so the minimum value of the function is 4. The domain is all real numbers. The range is all reals greater than or equal to the minimum value, that is $\{f(x) \mid f(x) \geq 4\}$.

b. $f(x) = 100 - 2x - x^2$

For this function, $a = -1$ and $b = -2$.

Since $a < 0$, the graph opens down, and the function has a maximum value.

The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{-2}{2(-1)} = -1$.

Evaluate the function at $x = -1$ to find the maximum value.

$f(-1) = 100 - 2(-1) - (-1)^2 = 101$, so the maximum value of the function is 101. The domain is all real numbers. The range is all reals less than or equal to the maximum value, that is $\{f(x) \mid f(x) \leq 101\}$.

Exercises

Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

1. $f(x) = 2x^2 - x + 10$
   min., $9\frac{7}{8}$; all reals; $\left\{f(x) \mid f(x) \geq 9\frac{7}{8}\right\}$
2. $f(x) = x^2 + 4x - 7$
   min., -11; all reals; $\{f(x) \mid f(x) \geq -11\}$
3. $f(x) = 3x^2 - 3x + 1$
   min., $\frac{1}{4}$; all reals; $\left\{f(x) \mid f(x) \geq \frac{1}{4}\right\}$
4. $f(x) = x^2 + 5x + 2$
   min., $-\frac{17}{4}$; all reals; $\left\{f(x) \mid f(x) \geq -\frac{17}{4}\right\}$
5. $f(x) = 20 + 6x - x^2$
   max., 29; all reals; $\{f(x) \mid f(x) \leq 29\}$
6. $f(x) = 4x^2 + x + 3$
   min., $2\frac{15}{16}$; all reals; $\left\{f(x) \mid f(x) \geq 2\frac{15}{16}\right\}$
7. $f(x) = -x^2 - 4x + 10$
   max., 14; all reals; $\{f(x) \mid f(x) \leq 14\}$
8. $f(x) = x^2 - 10x + 5$
   min., -20; all reals; $\{f(x) \mid f(x) \geq -20\}$
9. $f(x) = -6x^2 + 12x + 21$
   max., 27; all reals; $\{f(x) \mid f(x) \leq 27\}$

NAME _____ DATE _____ PERIOD _____

4-1 Skills Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

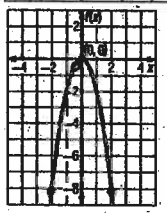
a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

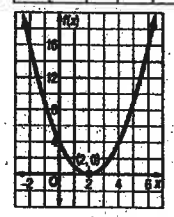
1. $f(x) = -2x^2$
   0; $x = 0$; 0

x	−2	−1	0	1	2
$f(x)$	−8	−2	0	−2	−2



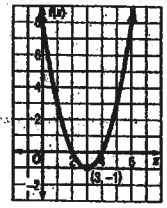
2. $f(x) = x^2 - 4x + 4$
   4; $x = 2$; 2

x	−2	0	2	4	6
$f(x)$	16	4	0	4	16



3. $f(x) = x^2 - 6x + 8$
   8; $x = 3$; 3

x	0	2	3	4	6
$f(x)$	8	0	−1	0	8



Determine whether each function has a maximum or a minimum value, and find that value. Then state the domain and range of the function.

4. $f(x) = 6x^2$
   min.; 0; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq 0\}$
5. $f(x) = -8x^2$
   max.; 0; D = {all real numbers}; R = $\{f(x) \mid f(x) \leq 0\}$
6. $f(x) = x^2 + 2x$
   min.; −1; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq -1\}$
7. $f(x) = -2x^2 + 4x - 3$
   max.; −1; D = {all real numbers}; R = $\{f(x) \mid f(x) \leq -1\}$
8. $f(x) = 3x^2 + 12x + 3$
   min.; −9; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq -9\}$
9. $f(x) = 2x^2 + 4x + 1$
   min.; −1; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq -1\}$
10. $f(x) = 3x^2$
    min.; 0; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq 0\}$
11. $f(x) = x^2 + 1$
    min.; 1; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq 1\}$
12. $f(x) = -x^2 + 6x - 15$
    max.; −6; D = {all real numbers}; R = $\{f(x) \mid f(x) \leq -6\}$
13. $f(x) = 2x^2 - 11$
    min.; −11; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq -11\}$
14. $f(x) = x^2 - 10x + 5$
    min.; −20; D = {all real numbers}; R = $\{f(x) \mid f(x) \geq -20\}$
15. $f(x) = -2x^2 + 8x + 7$
    max.; 15; D = {all real numbers}; R = $\{f(x) \mid f(x) \leq 15\}$

NAME _____ DATE _____ PERIOD _____

4-1 Practice

Graphing Quadratic Functions

Complete parts a–c for each quadratic function.

a. **Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.**

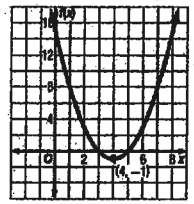
b. **Make a table of values that includes the vertex.**

c. **Use this information to graph the function.**

1. $f(x) = x^2 - 8x + 15$

15; $x = 4$; 4

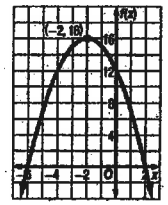
x	0	2	4	6	8
f(x)	15	3	−1	3	15



2. $f(x) = -x^2 - 4x + 12$

12; $x = -2$; −2

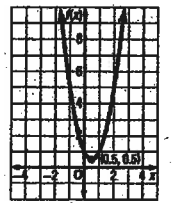
x	−6	−4	−2	0	2
f(x)	0	12	16	12	0



3. $f(x) = 2x^2 - 2x + 1$

1; $x = 0.5$; 0.5

x	−1	0	0.5	1	2
f(x)	5	1	0.5	1	5



Determine whether each function has a *maximum* or *minimum* value, and find that value. Then state the domain and range of the function.

4. $f(x) = x^2 + 2x - 8$

min.; −9; all reals; $\{f(x) \mid f(x) \geq -9\}$

5. $f(x) = x^2 - 6x + 14$

min.; 5; all reals; $\{f(x) \mid f(x) \geq 5\}$

6. $v(x) = -x^2 + 14x - 57$

max.; −8; all reals; $\{f(x) \mid f(x) \leq -8\}$

7. $f(x) = 2x^2 + 4x - 6$

min.; −8; all reals; $\{f(x) \mid f(x) \geq -8\}$

8. $f(x) = -x^2 + 4x - 1$

max.; 3; all reals; $\{f(x) \mid f(x) \leq 3\}$

9. $f(x) = -\frac{2}{3}x^2 + 8x - 24$

max.; 0; all reals; $\{f(x) \mid f(x) \leq 0\}$

10. **GRAVITATION** From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height $h(t)$ of the ball t seconds after Susan throws it is given by $h(t) = -16t^2 + 32t + 4$. For $t \geq 0$, find the maximum height reached by the ball and the time that this height is reached. **20 ft; 1 s**

11. **HEALTH CLUBS** Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price.

a. What price should the club charge to maximize the income from the aerobics classes?
$45

b. What is the maximum income the SportsTime Athletic Club can expect to make?
$2025

NAME _____ DATE _____ PERIOD _____

4-1 Word Problem Practice

Graphing Quadratic Functions

1. **TRAJECTORIES** A cannonball is launched from a cannon on the wall of Fort Chambly, Quebec. If the path of the cannonball is traced on a piece of graph paper aligned so that the cannon is situated on the *y*-axis, the equation that describes the path is

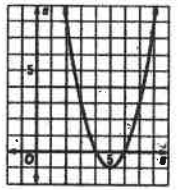
$$y = -\frac{1}{1600}x^2 + \frac{1}{2}x + 20,$$

where x is the horizontal distance from the cliff and y is the vertical distance above the ground in feet. How high above the ground is the cannon? **20 ft**

2. **TICKETING** The manager of a symphony computes that the symphony will earn $-40P^2 + 1100P$ dollars per concert if they charge P dollars for tickets. What ticket price should the symphony charge in order to maximize its profits? **$13.75**

3. **ARCHES** An architect decides to use a parabolic arch for the main entrance of a science museum. In one of his plans, the top edge of the arch is described by the graph of $y = -\frac{1}{4}x^2 + \frac{5}{2}x + 15$. What are the coordinates of the vertex of this parabola? **(5, 21.25)**

4. **FRAMING** A frame company offers a line of square frames. If the side length of the frame is s, then the area of the opening in the frame is given by the function $a(s) = s^2 - 10s + 24$. Graph $a(s)$.



5. **WALKING** Canal Street and Walker Street are perpendicular to each other. Evita is driving south on Canal Street and is currently 5 miles north of the intersection with Walker Street. Jack is at the intersection of Canal and Walker Streets and heading east on Walker. Jack and Evita are both driving 30 miles per hour.

a. When Jack is x miles east of the intersection, where is Evita?
$5 - x$ mi north of the intersection

b. The distance between Jack and Evita is given by the formula $\sqrt{x^2 + (5 - x)^2}$. For what value of x are Jack and Evita at their closest? (Hint: Minimize the square of the distance.)
$x = 2.5$

c. What is the distance of closest approach?
$\frac{5\sqrt{2}}{2}$ mi

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4-2 Study Guide and Intervention *(continued)*

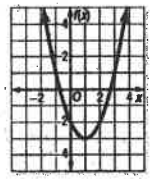
Solving Quadratic Equations by Graphing

Estimate Solutions Often, you may not be able to find exact solutions to quadratic equations by graphing. But you can use the graph to estimate solutions.

Example **Solve $x^2 - 2x - 2 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.**

The equation of the axis of symmetry of the related function is $x = \frac{-2}{2(1)} = 1$, so the vertex has x-coordinate 1. Make a table of values.

x	−1	0	1	2	3
$f(x)$	1	−2	−3	−2	1

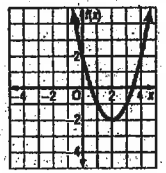


The x-intercepts of the graph are between 2 and 3 and between 0 and −1. So one solution is between 2 and 3, and the other solution is between 0 and −1.

Exercises

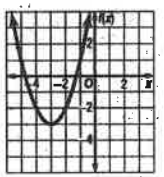
Solve the equations. If exact roots cannot be found, state the consecutive integers between which the roots are located.

1. $x^2 - 4x + 2 = 0$



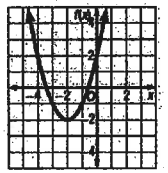
between 0 and 1; between 3 and 4

2. $x^2 + 6x + 6 = 0$



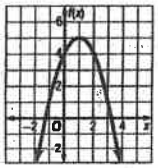
between −2 and −1; between −5 and −4

3. $x^2 + 4x + 2 = 0$



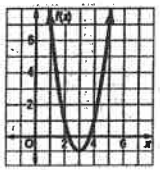
between −1 and 0; between −4 and −3

4. $-x^2 + 2x + 4 = 0$



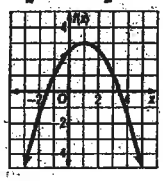
between 3 and 4; between −2 and −1

5. $2x^2 - 12x + 17 = 0$



between 2 and 3; between 3 and 4

6. $-\frac{1}{2}x^2 + x + \frac{5}{2} = 0$



between −2 and −1; between 3 and 4

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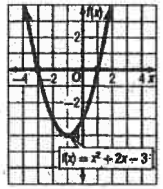
4-2 Skills Practice

Solving Quadratic Equations By Graphing

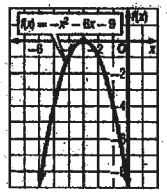
Use the related graph of each equation to determine its solutions.

1. $x^2 + 2x - 3 = 0$



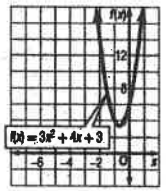
−3, 1

2. $-x^2 - 6x - 9 = 0$



−3

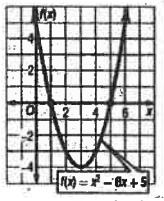
3. $3x^2 + 4x + 3 = 0$



no real solutions

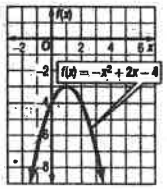
Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $x^2 - 6x + 5 = 0$



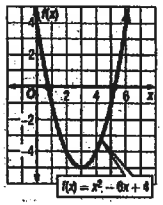
1, 5

5. $-x^2 + 2x - 4 = 0$



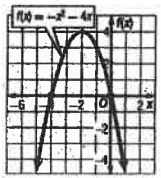
no real solutions

6. $x^2 - 6x + 4 = 0$



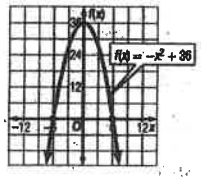
between 0 and 1; between 5 and 6

7. $-x^2 - 4x = 0$



0, −4

8. $-x^2 + 36 = 0$



−6, 6

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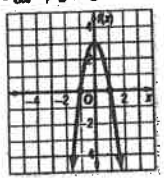
4-2 Practice

Solving Quadratic Equations By Graphing

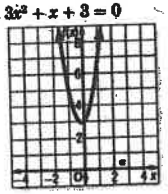
Use the related graph of each equation to determine its solutions.

1. $-3x^2 + 3 = 0$



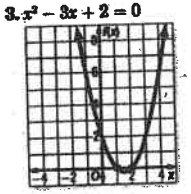
$-1, 1$

2. $3x^2 + x + 3 = 0$



no real solutions

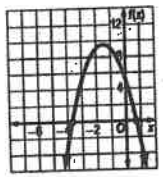
3. $x^2 - 3x + 2 = 0$



$1, 2$

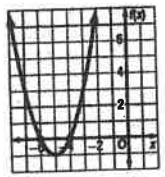
Solve each equation. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $-2x^2 - 6x + 5 = 0$



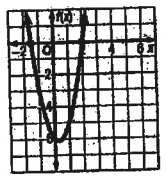
between 0 and 1; between -4 and -3

5. $x^2 + 10x + 24 = 0$



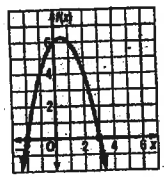
$-6, -4$

6. $2x^2 - x - 6 = 0$



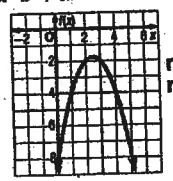
$-1.5, 2$

7. $-x^2 + x + 6 = 0$



$3, -2$

8. $-x^2 + 5x - 8 = 0$



no such real numbers exist

9. **GRAVITY** Use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

   a. Marta throws a baseball with an initial upward velocity of 60 feet per second. Ignoring Marta's height, how long after she releases the ball will it hit the ground?
   3.75 s

   b. A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected? **15 s**

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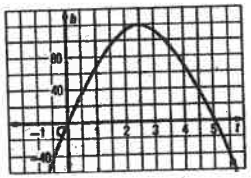
4-2 Word Problem Practice

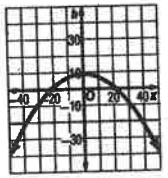
Solving Quadratic Equations by Graphing

1. **TRAJECTORIES** David threw a baseball into the air. The function of the height of the baseball in feet is $h = 80t - 16t^2$, where t represents the time in seconds after the ball was thrown. Use this graph of the function to determine how long it took for the ball to fall back to the ground.



5 seconds

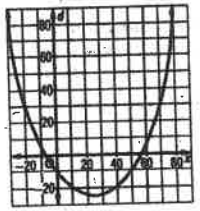
2. **BRIDGES** In 1895, a brick arch railway bridge was built on North Avenue in Baltimore, Maryland. The arch is described by the equation $h = 9 - \frac{1}{50}x^2$, where h is the height in yards and x is the distance in yards from the center of the bridge. Graph this equation and describe, to the nearest yard, where the bridge touches the ground.



21 yards from the center of the bridge on either side at (−21, 0) and (21, 0)

3. **LOGIC** Wilma is thinking of two numbers. The sum is 2 and the product is −24. Use a quadratic equation to find the two numbers.
6 and −4

4. **RADIO TELESCOPES** The cross-section of a large radio telescope is a parabola. The dish is set into the ground. The equation that describes the cross-section is $d = \frac{2}{75}x^2 - \frac{4}{3}x - \frac{32}{3}$, where d gives the depth of the dish below ground and x is the distance from the control center, both in meters. If the dish does not extend above the ground level, what is the diameter of the dish? Solve by graphing.



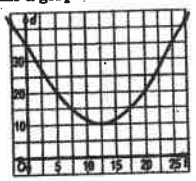
64 m

5. **BOATS** The distance between two boats is

$$d = \sqrt{t^2 - 10t + 35},$$

where d is distance in meters and t is time in seconds.

   a. Make a graph of d^2 versus t.



   b. Do the boats ever collide?
   No

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4-3 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Factoring

Solve Equations by Factoring When you use factoring to solve a quadratic equation, you use the following property.

Zero Product Property	For any real numbers a and b, if $ab = 0$, then either $a = 0$ or $b = 0$, or both a and $b = 0$.

Example **Solve each equation by factoring.**

a. $3x^2 = 15x$

$3x^2 = 15x$ Original equation
$3x^2 - 15x = 0$ Subtract $15x$ from both sides.
$3x(x - 5) = 0$ Factor the binomial.
$3x = 0$ or $x - 5 = 0$ Zero Product Property
$x = 0$ or $x = 5$ Solve each equation.

The solution set is $\{0, 5\}$.

b. $4x^2 - 5x = 21$

$4x^2 - 5x = 21$ Original equation
$4x^2 - 5x - 21 = 0$ Subtract 21 from both sides.
$(4x + 7)(x - 3) = 0$ Factor the trinomial.
$4x + 7 = 0$ or $x - 3 = 0$ Zero Product Property
$x = -\frac{7}{4}$ or $x = 3$ Solve each equation.

The solution set is $\left\{-\frac{7}{4}, 3\right\}$.

Exercises

Solve each equation by factoring.

1. $6x^2 - 2x = 0$ $\left\{0, \frac{1}{3}\right\}$
2. $x^2 = 7x$ $\{0, 7\}$
3. $20x^2 = -25x$ $\left\{0, -\frac{5}{4}\right\}$
4. $6x^2 = 7x$ $\left\{0, \frac{7}{6}\right\}$
5. $6x^2 - 27x = 0$ $\left\{0, \frac{9}{2}\right\}$
6. $12x^2 - 8x = 0$ $\left\{0, \frac{2}{3}\right\}$
7. $x^2 + x - 30 = 0$ $\{5, -6\}$
8. $2x^2 - x - 3 = 0$ $\left\{\frac{3}{2}, -1\right\}$
9. $x^2 + 14x + 33 = 0$ $\{-11, -3\}$
10. $4x^2 + 27x - 7 = 0$ $\left\{\frac{1}{4}, -7\right\}$
11. $3x^2 + 29x - 10 = 0$ $\left\{-10, \frac{1}{3}\right\}$
12. $6x^2 - 5x - 4 = 0$ $\left\{-\frac{1}{2}, \frac{4}{3}\right\}$
13. $12x^2 - 8x + 1 = 0$ $\left\{\frac{1}{6}, \frac{1}{2}\right\}$
14. $5x^2 + 28x - 12 = 0$ $\left\{\frac{2}{5}, -6\right\}$
15. $2x^2 - 250x + 5000 = 0$ $\{100, 25\}$
16. $2x^2 - 11x - 40 = 0$ $\left\{8, -\frac{5}{2}\right\}$
17. $2x^2 + 21x - 11 = 0$ $\left\{-11, \frac{1}{2}\right\}$
18. $3x^2 + 2x - 21 = 0$ $\left\{\frac{7}{3}, -3\right\}$
19. $8x^2 - 14x + 3 = 0$ $\left\{\frac{3}{2}, \frac{1}{4}\right\}$
20. $6x^2 + 11x - 2 = 0$ $\left\{-2, \frac{1}{6}\right\}$
21. $5x^2 + 17x - 12 = 0$ $\left\{\frac{3}{5}, -4\right\}$
22. $12x^2 + 25x + 12 = 0$ $\left\{-\frac{4}{3}, -\frac{3}{4}\right\}$
23. $12x^2 + 18x + 6 = 0$ $\left\{-\frac{1}{2}, -1\right\}$
24. $7x^2 - 36x + 5 = 0$ $\left\{\frac{1}{7}, 5\right\}$

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4-3 Skills Practice

Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1. 1, 4 $x^2 - 5x + 4 = 0$
2. 6, −9 $x^2 + 3x - 54 = 0$
3. −2, −5 $x^2 + 7x + 10 = 0$
4. 0, 7 $x^2 - 7x = 0$
5. $-\frac{1}{3}$, −3 $3x^2 + 10x + 3 = 0$
6. $-\frac{1}{2}, \frac{3}{4}$ $8x^2 - 2x - 3 = 0$

Factor each polynomial.

7. $m^2 + 7m - 18$ $(m - 2)(m + 9)$
8. $2x^2 - 3x - 5$ $(2x - 5)(x + 1)$
9. $4z^2 + 4z - 15$ $(2z + 5)(2z - 3)$
10. $4p^2 + 4p - 24$ $4(p - 2)(p + 3)$
11. $3y^2 + 21y + 36$ $3(y + 4)(y + 3)$
12. $c^2 - 100$ $(c + 10)(c - 10)$

Solve each equation by factoring.

13. $x^2 = 64$ $\{-8, 8\}$
14. $x^2 - 100 = 0$ $\{10, -10\}$
15. $x^2 - 3x + 2 = 0$ $\{1, 2\}$
16. $x^2 - 4x + 3 = 0$ $\{1, 3\}$
17. $x^2 + 2x - 3 = 0$ $\{1, -3\}$
18. $x^2 - 3x - 10 = 0$ $\{5, -2\}$
19. $x^2 - 6x + 5 = 0$ $\{1, 5\}$
20. $x^2 - 9x = 0$ $\{0, 9\}$
21. $x^2 - 4x = 21$ $\{-3, 7\}$
22. $2x^2 + 5x - 3 = 0$ $\left\{\frac{1}{2}, -3\right\}$
23. $4x^2 + 5x - 6 = 0$ $\left\{\frac{3}{4}, -2\right\}$
24. $3x^2 - 13x - 10 = 0$ $\left\{-\frac{2}{3}, 5\right\}$

25. **NUMBER THEORY** Find two consecutive integers whose product is 272. **16, 17 or −16, −17**

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4-3 Practice

Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

1. 7, 2
   $x^2 - 9x + 14 = 0$
2. 0, 3
   $x^2 - 3x = 0$
3. −5, 8
   $x^2 - 3x - 40 = 0$
4. −7, −8
   $x^2 + 15x + 56 = 0$
5. −6, −3
   $x^2 + 9x + 18 = 0$
6. 3, −4
   $x^2 + x - 12 = 0$
7. $1, \frac{1}{2}$
   $2x^2 - 3x + 1 = 0$
8. $\frac{1}{3}, 2$
   $3x^2 - 7x + 2 = 0$
9. $0, -\frac{7}{2}$
   $2x^2 + 7x = 0$

Factor each polynomial.

10. $r^3 + 3r^2 - 54r$
    $r(r + 9)(r - 6)$
11. $8a^2 + 2a - 6$
    $2(4a - 3)(a + 1)$
12. $c^2 - 49$
    $(c - 7)(c + 7)$
13. $x^3 + 8$
    $(x + 2)(x^2 - 2x + 4)$
14. $16r^3 - 169$
    $(4r + 13)(4r - 13)$
15. $b^4 - 81$
    $(b^2 + 9)(b + 3)(b - 3)$

Solve each equation by factoring.

16. $x^2 - 4x - 12 = 0$ {6, −2}
17. $x^2 - 16x + 64 = 0$ {8}
18. $x^2 - 6x + 8 = 0$ {2, 4}
19. $x^2 + 3x + 2 = 0$ {−2, −1}
20. $x^2 - 4x = 0$ {0, 4}
21. $7x^2 = 4x$ $\left\{0, \frac{4}{7}\right\}$
22. $10x^2 = 9x$ $\left\{0, \frac{9}{10}\right\}$
23. $x^2 = 2x + 99$ {−9, 11}
24. $x^2 + 12x = -36$ {−6}
25. $5x^2 - 35x + 60 = 0$ {3, 4}
26. $36x^2 = 25$ $\left\{\frac{5}{6}, -\frac{5}{6}\right\}$
27. $2x^2 - 8x - 90 = 0$ {9, −5}

28. **NUMBER THEORY** Find two consecutive even positive integers whose product is 624.
    24, 26
29. **NUMBER THEORY** Find two consecutive odd positive integers whose product is 323.
    17, 19
30. **GEOMETRY** The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet. 7 ft by 9 ft
31. **PHOTOGRAPHY** The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced? 2 in.

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4-3 Word Problem Practice

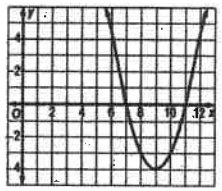
Solving Quadratic Equations by Factoring

1. **FLASHLIGHTS** When Dora shines her flashlight on the wall at a certain angle, the edge of the lit area is in the shape of a parabola. The equation of the parabola is $y = 2x^2 + 2x - 60$. Factor this quadratic equation.
   $2(x - 5)(x + 6)$

2. **SIGNS** David was looking through an old algebra book and came across this equation.
   $x^2 \_\_ 6x + 8 = 0$
   The sign in front of the 6 was blotted out. How does the missing sign depend on the signs of the roots?
   The missing sign is the opposite of the sign of the two roots, because their product is a positive number, 8.

3. **ART** The area in square inches of the drawing *Maisons prés de la mer* by Claude Monet is approximated by the equation $y = x^2 - 23x + 130$. Factor the equation to find the two roots, which are equal to the approximate length and width of the drawing.
   10 inches by 13 inches

4. **PROGRAMMING** Ray is a computer programmer. He needs to find the quadratic function of this graph for an algorithm related to a game involving dice. Provide such a function.



   $f(x) = x^2 - 18x + 77$

5. **ANIMATION** A computer graphics animator would like to make a realistic simulation of a tossed ball. The animator wants the ball to follow the parabolic trajectory represented by the quadratic equation $f(x) = -0.2(x + 5)(x - 5)$.
   a. What are the solutions of $f(x) = 0$?
      $x = -5$ or $x = 5$
   b. Write $f(x)$ in standard form.
      $f(x) = -0.2x^2 + 5$
   c. If the animator changes the equation to $f(x) = -0.2x^2 + 20$, what are the solutions of $f(x) = 0$?
      $x = -10$ or $x = 10$

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4-4 Skills Practice

Complex Numbers

Simplify.

1. $\sqrt{99}$ $3\sqrt{11}$
2. $\sqrt{\frac{27}{49}}$ $\frac{3\sqrt{3}}{7}$
3. $\sqrt{52x^5y^5}$ $2|x|y^2\sqrt{13xy}$
4. $\sqrt{-108x^7}$ $6i|x^3|\sqrt{3x}$
5. $\sqrt{-81x^6}$ $9|x^3|i$
6. $\sqrt{-23}\cdot\sqrt{-46}$ $-23\sqrt{2}$
7. $(3i)(-2i)(5i)$ $30i$
8. i^{11} $-i$
9. i^{65} i
10. $(7-8i)+(-12-4i)$ $-5-12i$
11. $(-3+5i)+(18-7i)$ $15-2i$
12. $(10-4i)-(7+3i)$ $3-7i$
13. $(7-6i)(2-3i)$ $-4-33i$
14. $(3+4i)(3-4i)$ 25
15. $\frac{8-6i}{3i}$ $-2-\frac{8}{3}i$
16. $\frac{3i}{4+2i}$ $\frac{3}{10}+\frac{3}{5}i$

Solve each equation.

17. $3x^2+3=0$ $\pm i$
18. $5x^2+125=0$ $\pm 5i$
19. $4x^2+20=0$ $\pm i\sqrt{5}$
20. $-x^2-16=0$ $\pm 4i$
21. $x^2+18=0$ $\pm 3i\sqrt{2}$
22. $8x^2+96=0$ $\pm 2i\sqrt{3}$

Find the values of ℓ and m that make each equation true.

23. $20-12i=5\ell+(4m)i$ $4, -3$
24. $\ell-16i=3-(2m)i$ $3, 8$
25. $(4+\ell)+(2m)i=9+14i$ $5, 7$
26. $(3-m)+(7\ell-14)i=1+7i$ $3, 2$

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4-4 Practice

Complex Numbers

Simplify.

1. $\sqrt{-36}$ $6i$
2. $\sqrt{-8}\cdot\sqrt{-32}$ -16
3. $\sqrt{-15}\cdot\sqrt{-25}$ $-5\sqrt{15}$
4. $(-3i)(4i)(-5i)$ $-60i$
5. $(7i)^2(6i)$ $-294i$
6. i^{42} -1
7. i^{55} $-i$
8. i^{80} 1
9. $(5-2i)+(-13-8i)$ $-8-10i$
10. $(7-6i)+(9+11i)$ $16+5i$
11. $(-12+48i)+(15+21i)$ $3+69i$
12. $(10+15i)-(48-30i)$ $-38+45i$
13. $(28-4i)-(10-30i)$ $18+26i$
14. $(6-4i)(6+4i)$ 52
15. $(8-11i)(8-11i)$ $-57-176i$
16. $(4+3i)(2-5i)$ $23-14i$
17. $(7+2i)(9-6i)$ $75-24i$
18. $\frac{6+5i}{-2i}$ $-\frac{5}{2}+3i$
19. $\frac{2}{7-8i}$ $\frac{14}{113}+\frac{16}{113}i$
20. $\frac{3-i}{2-i}$ $\frac{7}{5}+\frac{1}{5}i$
21. $\frac{2-4i}{1+3i}$ $-1-i$

Solve each equation.

22. $5n^2+35=0$ $\pm i\sqrt{7}$
23. $2m^2+10=0$ $\pm i\sqrt{5}$
24. $4m^2+76=0$ $\pm i\sqrt{19}$
25. $-2m^2-6=0$ $\pm i\sqrt{3}$
26. $-5m^2-65=0$ $\pm i\sqrt{13}$
27. $\frac{3}{4}x^2+12=0$ $\pm 4i$

Find the values of ℓ and m that make each equation true.

28. $15-28i=3\ell+(4m)i$ $5, -7$
29. $(6-\ell)+(3m)i=-12+27i$ $18, 9$
30. $(3\ell+4)+(3-m)i=16-3i$ $4, 6$
31. $(7+m)+(4\ell-10)i=3-6i$ $1, -4$

32. **ELECTRICITY** The impedance in one part of a series circuit is $1+3j$ ohms and the impedance in another part of the circuit is $7-5j$ ohms. Add these complex numbers to find the total impedance in the circuit. $8-2j$ ohms

33. **ELECTRICITY** Using the formula $E=IZ$, find the voltage E in a circuit when the current I is $3-j$ amps and the impedance Z is $3+2j$ ohms. $11+3j$ volts