

# AP CALCULUS

## Stuff you **MUST** know Cold

\* means topic only on BC

<p><b>Curve sketching and analysis</b>  <math>y = f(x)</math> must be continuous at each:  critical point: <math>\frac{dy}{dx} = 0</math> or <u>undefined</u>  and look out for endpoints  local minimum:  <math>\frac{dy}{dx}</math> goes <math>(-, 0, +)</math> or <math>(-, \text{und}, +)</math> or <math>\frac{d^2y}{dx^2} &gt; 0</math>  local maximum:  <math>\frac{dy}{dx}</math> goes <math>(+, 0, -)</math> or <math>(+, \text{und}, -)</math> or <math>\frac{d^2y}{dx^2} &lt; 0</math>  point of inflection: concavity changes  <math>\frac{d^2y}{dx^2}</math> goes from <math>(+, 0, -)</math>, <math>(-, 0, +)</math>,  <math>(+, \text{und}, -)</math>, or <math>(-, \text{und}, +)</math></p>	<p><b>Differentiation Rules</b>  <b>Chain Rule</b>  <math>\frac{d}{dx}[f(u)] = f'(u) \frac{du}{dx}</math> OR <math>\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}</math>  <b>Product Rule</b>  <math>\frac{d}{dx}(uv) = \frac{du}{dx}v + u \frac{dv}{dx}</math> OR <math>u'v + uv'</math>  <b>Quotient Rule</b>  <math>\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u \frac{dv}{dx}}{v^2}</math> OR <math>\frac{u'v - uv'}{v^2}</math></p>	<p><b>Approx. Methods for Integration</b>  <b>Trapezoidal Rule</b>  <math>\int_a^b f(x)dx = \frac{1}{2} \frac{b-a}{n} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]</math>  <b>Simpson's Rule</b>  <math>\int_a^b f(x)dx = \frac{1}{3} \Delta x [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]</math></p>
<p><b>Basic Derivatives</b>  <math>\frac{d}{dx}(x^n) = nx^{n-1}</math>  <math>\frac{d}{dx}(\sin x) = \cos x</math>  <math>\frac{d}{dx}(\cos x) = -\sin x</math>  <math>\frac{d}{dx}(\tan x) = \sec^2 x</math>  <math>\frac{d}{dx}(\cot x) = -\csc^2 x</math>  <math>\frac{d}{dx}(\sec x) = \sec x \tan x</math>  <math>\frac{d}{dx}(\csc x) = -\csc x \cot x</math>  <math>\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}</math>  <math>\frac{d}{dx}(e^u) = e^u \frac{du}{dx}</math>  where <math>u</math> is a function of <math>x</math>,  and <math>a</math> is a constant.</p>	<p><b>“PLUS A CONSTANT”</b>  <b>The Fundamental Theorem of Calculus</b>  <math>\int_a^b f(x)dx = F(b) - F(a)</math>  where <math>F'(x) = f(x)</math>  <b>Corollary to FTC</b>  <math>\frac{d}{dx} \int_{a(x)}^{b(x)} f(t)dt = f(b(x))b'(x) - f(a(x))a'(x)</math>  <b>Intermediate Value Theorem</b>  If the function <math>f(x)</math> is continuous on <math>[a, b]</math>,  and <math>y</math> is a number between <math>f(a)</math> and <math>f(b)</math>,  then there exists at least one number <math>x = c</math>  in the open interval <math>(a, b)</math> such that  <math>f(c) = y</math>.</p>	<p><b>Theorem of the Mean Value</b>  <b>i.e. AVERAGE VALUE</b>  If the function <math>f(x)</math> is continuous on <math>[a, b]</math>  and the first derivative exists on the  interval <math>(a, b)</math>, then there exists a number  <math>x = c</math> on <math>(a, b)</math> such that  <math display="block">f(c) = \frac{\int_a^b f(x)dx}{(b-a)}</math> This value <math>f(c)</math> is the “average value” of  the function on the interval <math>[a, b]</math>.  <b>Solids of Revolution</b> and friends  <b>Disk Method</b>  <math>V = \pi \int_{x=a}^{x=b} [R(x)]^2 dx</math>  <b>Washer Method</b>  <math>V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx</math>  General volume equation (not rotated)  <math>V = \int_a^b \text{Area}(x) dx</math>  *Arc Length <math>L = \int_a^b \sqrt{1 + [f'(x)]^2} dx</math>  <math>= \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt</math></p>
<p><b>More Derivatives</b>  <math>\frac{d}{dx}\left(\sin^{-1} \frac{u}{a}\right) = \frac{1}{\sqrt{a^2 - u^2}} \frac{du}{dx}</math>  <math>\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1 - x^2}}</math>  <math>\frac{d}{dx}\left(\tan^{-1} \frac{u}{a}\right) = \frac{a}{a^2 + u^2} \cdot \frac{du}{dx}</math>  <math>\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1 + x^2}</math>  <math>\frac{d}{dx}\left(\sec^{-1} \frac{u}{a}\right) = \frac{a}{ u \sqrt{u^2 - a^2}} \cdot \frac{du}{dx}</math>  <math>\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{ x \sqrt{x^2 - 1}}</math>  <math>\frac{d}{dx}(a^{u(x)}) = a^{u(x)} \ln a \cdot \frac{du}{dx}</math>  <math>\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}</math></p>	<p><b>Mean Value Theorem</b>  If the function <math>f(x)</math> is continuous on <math>[a, b]</math>,  AND the first derivative exists on the  interval <math>(a, b)</math>, then there is at least one  number <math>x = c</math> in <math>(a, b)</math> such that  <math display="block">f'(c) = \frac{f(b) - f(a)}{b - a}</math>  <b>Rolle's Theorem</b>  If the function <math>f(x)</math> is continuous on <math>[a, b]</math>,  AND the first derivative exists on the  interval <math>(a, b)</math>, AND <math>f(a) = f(b)</math>, then there  is at least one number <math>x = c</math> in <math>(a, b)</math> such  that  <math display="block">f'(c) = 0</math></p>	<p><b>Distance, Velocity, and Acceleration</b>  velocity = <math>\frac{d}{dt}</math> (position)  acceleration = <math>\frac{d}{dt}</math> (velocity)  *velocity vector = <math>\left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle</math>  speed = <math> v  = \sqrt{(x')^2 + (y')^2}</math> *  displacement = <math>\int_{t_0}^{t_f} v dt</math>  distance = <math>\int_{\text{initial time}}^{\text{final time}}  v  dt</math>  <math>\int_{t_0}^{t_f} \sqrt{(x')^2 + (y')^2} dt</math> *  average velocity =  <math display="block">= \frac{\text{final position} - \text{initial position}}{\text{total time}}</math>  <math display="block">= \frac{\Delta x}{\Delta t}</math></p>

## BC TOPICS and important TRIG identities and values

<b>L'Hôpital's Rule</b> If $\frac{f(a)}{g(b)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ , then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$	<b>Slope of a Parametric equation</b> Given a $x(t)$ and a $y(t)$ the slope is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	<b>Values of Trigonometric Functions for Common Angles</b> <table><tr><th><math>\theta</math></th><th><math>\sin \theta</math></th><th><math>\cos \theta</math></th><th><math>\tan \theta</math></th></tr><tr><td><math>0^\circ</math></td><td>0</td><td>1</td><td>0</td></tr><tr><td><math>\frac{\pi}{6}</math></td><td><math>\frac{1}{2}</math></td><td><math>\frac{\sqrt{3}}{2}</math></td><td><math>\frac{\sqrt{3}}{3}</math></td></tr><tr><td><math>\frac{\pi}{4}</math></td><td><math>\frac{\sqrt{2}}{2}</math></td><td><math>\frac{\sqrt{2}}{2}</math></td><td>1</td></tr><tr><td><math>\frac{\pi}{3}</math></td><td><math>\frac{\sqrt{3}}{2}</math></td><td><math>\frac{1}{2}</math></td><td><math>\sqrt{3}</math></td></tr><tr><td><math>\frac{\pi}{2}</math></td><td>1</td><td>0</td><td>"<math>\infty</math>"</td></tr><tr><td><math>\pi</math></td><td>0</td><td>-1</td><td>0</td></tr></table> <p>Know both the <i>inverse trig</i> and the <i>trig</i> values. E.g. <math>\tan(\pi/4)=1</math> &amp; <math>\tan^{-1}(1)=\pi/4</math></p>	$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$	$0^\circ$	0	1	0	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\pi}{2}$	1	0	" $\infty$ "	$\pi$	0	-1	0
$\theta$	$\sin \theta$	$\cos \theta$	$\tan \theta$																											
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$\pi$	0	-1	0																											
<b>Euler's Method</b> If given that $\frac{dy}{dx} = f(x, y)$ and that the solution passes through $(x_o, y_o)$ , $y(x_o) = y_o$ $\vdots$ $y(x_n) = y(x_{n-1}) + f(x_{n-1}, y_{n-1}) \cdot \Delta x$ In other words: $x_{\text{new}} = x_{\text{old}} + \Delta x$ $y_{\text{new}} = y_{\text{old}} + \left. \frac{dy}{dx} \right _{(x_{\text{old}}, y_{\text{old}})} \cdot \Delta x$	<b>Polar Curve</b> For a polar curve $r(\theta)$ , the <b>AREA</b> inside a "leaf" is $\int_{\theta_1}^{\theta_2} \frac{1}{2} [r(\theta)]^2 d\theta$ where $\theta_1$ and $\theta_2$ are the "first" two times that $r = 0$ . The <b>SLOPE</b> of $r(\theta)$ at a given $\theta$ is $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta} [r(\theta) \sin \theta]}{\frac{d}{d\theta} [r(\theta) \cos \theta]}$	<b>Trig Identities</b> <i>Double Argument</i> $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ <i>Pythagorean</i> $\sin^2 x + \cos^2 x = 1$ (others are easily derivable by dividing by $\sin^2 x$ or $\cos^2 x$ ) $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$ <i>Reciprocal</i> $\sec x = \frac{1}{\cos x}$ or $\cos x \sec x = 1$ $\csc x = \frac{1}{\sin x}$ or $\sin x \csc x = 1$ <i>Odd-Even</i> $\sin(-x) = -\sin x$ (odd) $\cos(-x) = \cos x$ (even) <i>Some more handy INTEGRALS:</i> $\int \tan x dx = \ln \sec x  + C$ $= -\ln \cos x  + C$ $\int \sec x dx = \ln \sec x + \tan x  + C$																												
<b>Integration by Parts</b> $\int u dv = uv - \int v du$ <b>Integral of Log</b> Use IBP and let $u = \ln x$ (Recall $u=LIPET$ ) $\int \ln x dx = x \ln x - x + C$	<b>Ratio Test</b> The series $\sum_{k=0}^{\infty} a_k$ converges if $\lim_{k \rightarrow \infty} \left  \frac{a_{k+1}}{a_k} \right  < 1$ If the limit equal 1, you know nothing.																													
<b>Taylor Series</b> If the function $f$ is "smooth" at $x = a$ , then it can be approximated by the $n^{\text{th}}$ degree polynomial $f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$	<b>Lagrange Error Bound</b> If $P_n(x)$ is the $n^{\text{th}}$ degree Taylor polynomial of $f(x)$ about $c$ and $ f^{(n+1)}(t)  \leq M$ for all $t$ between $x$ and $c$ , then $ f(x) - P_n(x)  \leq \frac{M}{(n+1)!}  x - c ^{n+1}$																													
<b>Maclaurin Series</b> A Taylor Series about $x = 0$ is called Maclaurin. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$ $\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	<b>Alternating Series Error Bound</b> If $S_N = \sum_{k=1}^N (-1)^n a_n$ is the $N^{\text{th}}$ partial sum of a convergent alternating series, then $ S_{\infty} - S_N  \leq  a_{N+1} $ <b>Geometric Series</b> $a + ar + ar^2 + ar^3 + \dots + ar^{n-1} + \dots = \sum_{n=1}^{\infty} ar^{n-1}$ diverges if $ r  \geq 1$ ; converges to $\frac{a}{1-r}$ if $ r  < 1$																													