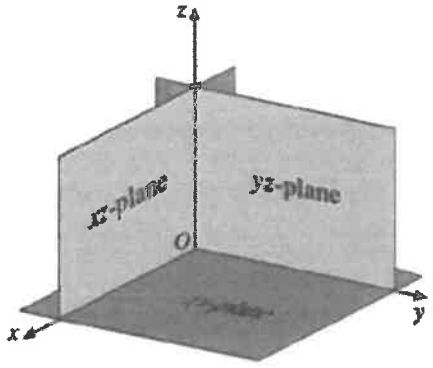
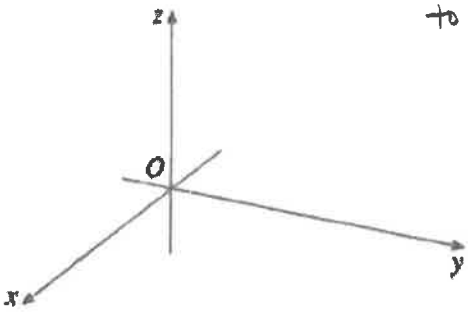
Coordinate planes

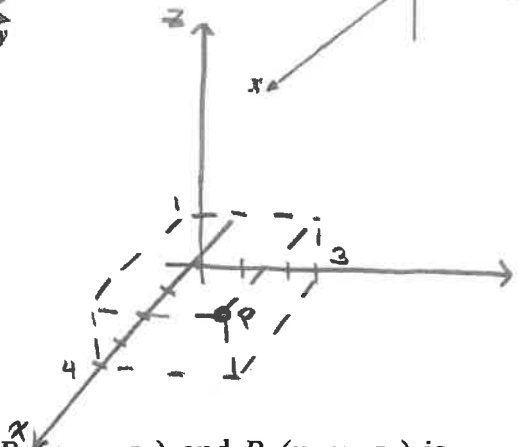


Coordinate axis

planes perpendicular to each other



1. Plot the point $P(4, 3, 1)$ (x, y, z)



Distance Formula

The distance between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is:

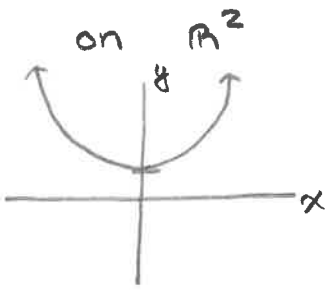
$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

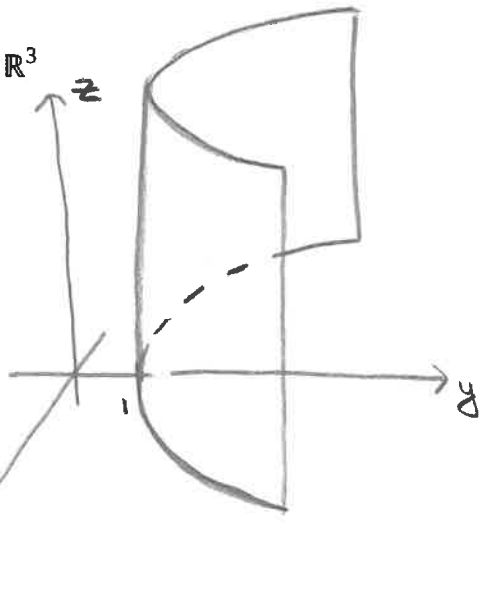
Cylindrical surfaces

An equation that only two of the variables x, y, and z represents a curve when graphed in $\mathbb{R}^2$ and a cylindrical surface when graphed in $\mathbb{R}^3$.

To graph the cylindrical surface, first graph the equation in the coordinate plane of the two variables and then translate that graph with respect to the axis of the missing variable.

2. Sketch the graph of the parabolic cylinder $y = x^2 + 1$ in $\mathbb{R}^3$

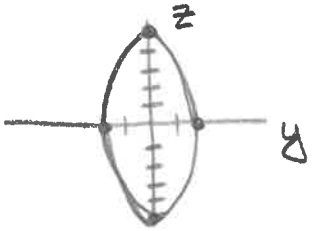


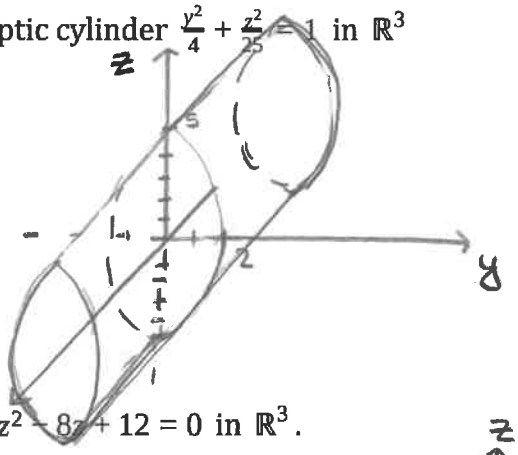


no matter what z is will be this graph

3. Sketch the graph of the elliptic cylinder $\frac{y^2}{4} + \frac{z^2}{25} = 1$ in $\mathbb{R}^3$

on $\mathbb{R}^2$

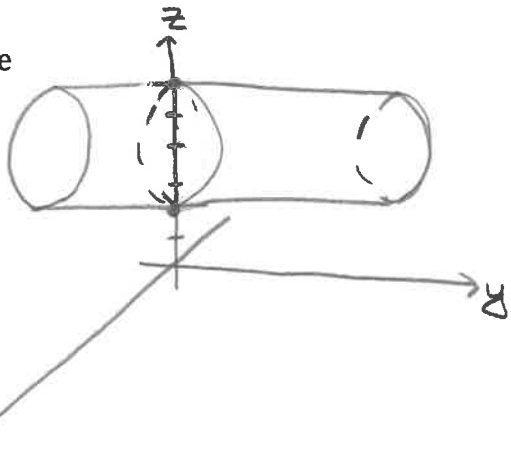




4. Let S be the graph of $x^2 + z^2 - 8z + 12 = 0$ in $\mathbb{R}^3$.

a. Describe S and sketch a graph of the surface

circular cylinder



$x^2 + z^2 - 8z = -12$

$(x-0)^2 + z^2 - 8z + 16 = -12 + 16$

$(x-0)^2 + (z-4)^2 = 4$

center $(0,4)$ $r = 2$

x, z

b. What is the intersection of S with the xz – plane?

↳ $y = 0$

circle $x^2 + (z-4)^2 = 4$

c. What is the intersection of S with the yz – plane?

↳ $x = 0$

2 lines

$0 + (z-4)^2 = 4$

$z - 4 = \pm 2$

$z = 4 \pm 2$

$z = 6, 2$

$x = 0$ $z = 6$ $y =$ any #

$x = 0$ $z = 2$ $y =$ any #

d. What is the intersection of S with the xy – plane?

none

check $z = 0$

$x^2 + (0-4)^2 = 4$

$x^2 + 16 = 4$

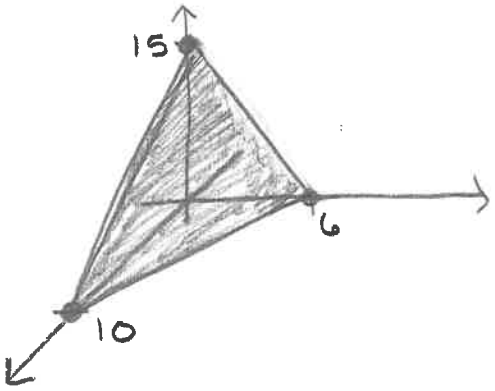
$x^2 = -12$

Plane

The equation of a plane is of the form $ax + by + cz = d$ where a, b, c, and d are constants.

5. Find the points where the plane $3x + 5y + 2z = 30$ intersects the coordinate axis. Sketch a graph of this plane.

x	y	z	(x, y, z)
0	0	15	(0, 0, 15)
0	6	0	(0, 6, 0)
10	0	0	(10, 0, 0)



Sphere

An equation of a sphere with center $C(h, k, l)$ and radius r is
$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$

6. Find an equation of a sphere with center at $(3, 4, -1)$ and radius of 7.

$$(x-3)^2 + (y-4)^2 + (z+1)^2 = 49$$

7. Use the sphere $(x - 3)^2 + (y - 4)^2 + (z + 7)^2 = 25$ to answer the following.

a. Find the intersection of the sphere and the xz coordinate plane.

$\hookrightarrow y = 0$

$$(x-3)^2 + (0-4)^2 + (z+7)^2 = 25$$

circle

$$(x-3)^2 + 16 + (z+7)^2 = 25$$

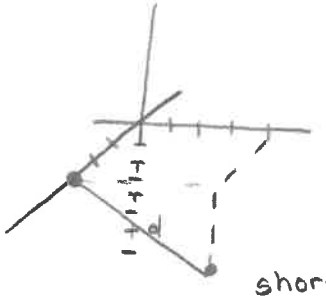
$$(x-3)^2 + (z+7)^2 = 9$$

b. What is the distance from the center of the sphere to the xz – plane?

center $(3, 4, -7)$ y controls how far away from xz - plane

4 units

c. How far is the center from the x-axis?



$$d = \sqrt{(3-3)^2 + (0-4)^2 + (0+7)^2}$$

$$= \sqrt{16 + 49}$$

$$= \sqrt{65}$$

shortest distance would be to $(3, 0, 0)$

8. Find the center and radius of the sphere $2x^2 + 2y^2 + 2z^2 + 8y - 6z = 4$.

$2x^2 + 2y^2 + 2z^2 + 8y - 6z = 4$

$x^2 + y^2 + z^2 + 4y - 3z = 2$

$(x-0)^2 + y^2 + 4y + z^2 - 3z = 2$

$(x-0)^2 + y^2 + 4y + 4 + z^2 - 3z + 9/4 = 2 + 4 + 9/4$

$(x-0)^2 + (y+2)^2 + (z - 3/2)^2 = 8.25$

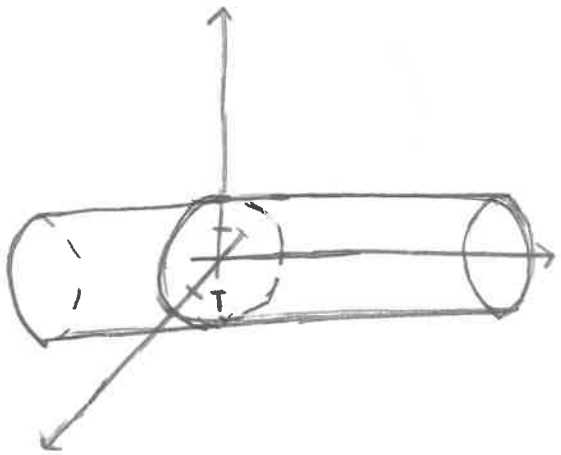
center $(0, -2, -3/2)$ $r = \sqrt{8.25}$

9. Describe the following region of $\mathbb{R}^3$ represented by the equations

$x^2 + z^2 = 10$, $y = 4$

$x^2 + z^2 = 10$ → circular cylinder centered $(0,0)$ $r = \sqrt{10}$

$y = 4$ → plane through $y = 4$



The circle in the plane $y = 4$ $x^2 + z^2 = 10$