The term vector is used to represent a quantity that has both magnitude and direction. We denote a vector by:

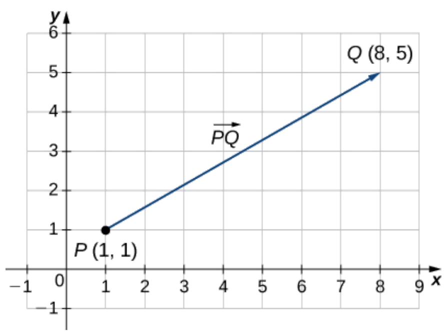
The zero vector, denoted **0**, has length zero and is the only vector that does not have a specific direction.

A two dimensional vector has the form $\mathbf{a} = \langle a_1, a_2 \rangle$ and a three dimensional vector has the form $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, where a_1, a_2 and a_3 are real numbers andare called components of the vector.

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

How does this formula make sense? Let's look at:



$P(1, 1)$ and $Q(8, 5)$

$\overrightarrow{PQ}$ we know would be $\langle 7, 4 \rangle$ so $\langle 8-1, 5-1 \rangle$

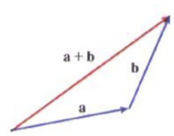
1. For the points, $A(1, 2, 8)$ and $B(4, 7, 2)$, find $\overrightarrow{AB}$ and $\overrightarrow{BA}$

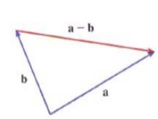
Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle, \mathbf{b} = \langle b_1, b_2, b_3 \rangle$ and c be a scalar, i.e. $c \in \mathbb{R}$.

Scalar Multiplication: $c\mathbf{a} = c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$

Length or Magnitude of $\mathbf{a}$ is $|\mathbf{a}| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$

Vector Addition: $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

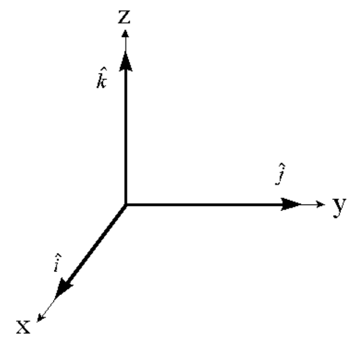




Vector Subtraction: $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

Two vectors are parallel if one vector is a scalar multiple of the other.
i.e. there exists $c \in \mathbb{R}$ such that $c\mathbf{a} = \mathbf{b}$

A vector of length 1 is called a unit vector. The vectors $\mathbf{i} = \langle 1, 0, 0 \rangle, \mathbf{j} = \langle 0, 1, 0 \rangle$, and $\mathbf{k} = \langle 0, 0, 1 \rangle$ are called the standard basis vectors for $\mathbb{R}^3$.



$\overrightarrow{A} = 3i + 2j + k = \langle 3, 2, 1 \rangle$ on $\mathbb{R}^3$ coordinate plane:

To find a unit vector in the same direction as $\mathbf{a}$, divide $\mathbf{a}$ by its magnitude. This process is called normalizing $\mathbf{a}$.

2. Find the following using the vectors $\mathbf{a} = \langle 1, 2, 4 \rangle$ and $\mathbf{c} = \langle 2, -4, 1 \rangle$
   a. $3\mathbf{a} - 2\mathbf{c}$

   b. Find a vector of length 3 in the opposite direction of $\mathbf{a}$

3. Consider points $A = (0, -1, 3)$, $B = (3, 1, 1)$, $C = (2, 2, 2)$, and $D = (3, 0, -1)$
   a. Compute the components of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$

   b. Compute $\overrightarrow{AB} - \overrightarrow{CD}$

c. Compute $\frac{1}{4}\overrightarrow{AB} + \overrightarrow{CD}$

d. Compute $|\overrightarrow{AB}|$

4. Let $\overrightarrow{u} = 2i + j + k$ and $\overrightarrow{v} = i + 4j - k$. Find $\overrightarrow{w} = \overrightarrow{u} - \overrightarrow{v}$ and find the unit vector $\overrightarrow{a}$ in the same direction as $\overrightarrow{w}$