The term vector is used to represent a quantity that has both magnitude and direction. We denote a vector by: bold letter or $\vec{a}$

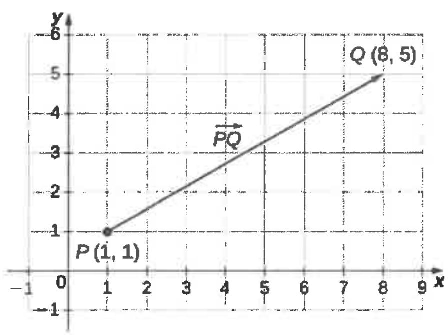
The zero vector, denoted **0**, has length zero and is the only vector that does not have a specific direction.

A two dimensional vector has the form $\mathbf{a} = \langle a_1, a_2 \rangle$ and a three dimensional vector has the form $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, where a_1, a_2 and a_3 are real numbers are called components of the vector.

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

How does this formula make sense? Let's look at:



$P(1,1)$ and $Q(8,5)$

$\overrightarrow{PQ}$ we know would be $\langle 7, 4 \rangle$ so $\langle 8-1, 5-1 \rangle$

1. For the points, $A(1,2,8)$ and $B(4,7,2)$, find $\overrightarrow{AB}$ and $\overrightarrow{BA}$

$\overrightarrow{AB} = \langle 4-1, 7-2, 2-8 \rangle = \langle 3, 5, -6 \rangle$

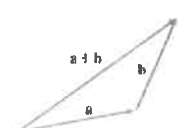
$\overrightarrow{BA} = \langle 1-4, 2-7, 8-2 \rangle = \langle -3, -5, 6 \rangle$

Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle, \mathbf{b} = \langle b_1, b_2, b_3 \rangle$ and c be a scalar, i.e. $c \in \mathbb{R}$.

Scalar Multiplication: $c\mathbf{a} = c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$

Length or Magnitude of $\mathbf{a}$ is $|\mathbf{a}| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$

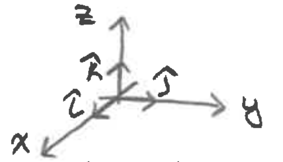
Vector Addition: $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$



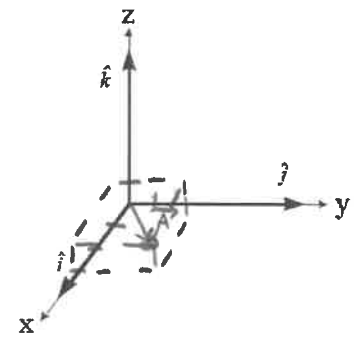
Vector Subtraction: $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

Two vectors are parallel if one vector is a scalar multiple of the other.
i.e. there exists $c \in \mathbb{R}$ such that $c\mathbf{a} = \mathbf{b}$

A vector of length 1 is called a unit vector. The vectors $\mathbf{i} = \langle 1,0,0\rangle, \mathbf{j} = \langle 0,1,0\rangle$, and $\mathbf{k} = \langle 0,0,1\rangle$ are called the standard basis vectors for $\mathbb{R}^3$.



$\vec{A} = 3i + 2j + k = \langle 3,2,1\rangle$ on $\mathbb{R}^3$ coordinate plane:



To find a unit vector in the same direction as $\mathbf{a}$, divide $\mathbf{a}$ by its magnitude. This process is called normalizing $\mathbf{a}$.

unit $= \dfrac{a}{|a|}$

2. Find the following using the vectors $\mathbf{a} = \langle 1,2,4\rangle$ and $\mathbf{c} = \langle 2,-4,1\rangle$
   a. $3\mathbf{a} - 2\mathbf{c}$

   $\langle 3, 6, 12\rangle \quad - \quad \langle 4, -8, 2\rangle$

   $3\vec{a} - 2\vec{c} = \boxed{\langle -1, 14, 10\rangle}$

   b. Find a vector of length 3 in the opposite direction of $\mathbf{a}$ * find unit vector first

   $|a| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$ unit $= \dfrac{1}{\sqrt{21}}\langle 1,2,4\rangle$

   answer → $\dfrac{-3}{\sqrt{21}}\langle 1,2,4\rangle = \boxed{\left\langle -3/\sqrt{21}, -6/\sqrt{21}, -12/\sqrt{21}\right\rangle}$

3. Consider points $A = (0,-1,3)$, $B = (3,1,1)$, $C = (2,2,2)$, and $D = (3,0,-1)$
   a. Compute the components of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$

   $\overrightarrow{AB} = \langle 3-0, 1-(-1), 1-3\rangle = \langle 3, 2, -2\rangle$

   $\overrightarrow{CD} = \langle 3-2, 0-2, -1-2\rangle = \langle 1, -2, -3\rangle$

   b. Compute $\overrightarrow{AB} - \overrightarrow{CD}$

   $\overrightarrow{AB} - \overrightarrow{CD} = \langle 3-1, 2-(-2), -2-(-3)\rangle = \langle 2, 4, 1\rangle$

c. Compute $\frac{1}{4}\overrightarrow{AB} + \overrightarrow{CD}$

$\frac{1}{4}\langle 3, 2, -2\rangle + \langle 1, -2, -3\rangle$

$\langle 3/4 + 1,\ 1/2 - 2,\ -1/2 - 3\rangle$

$\boxed{\langle 7/4,\ -3/2,\ -7/2\rangle}$

d. Compute $|\overrightarrow{AB}|$

$|\overrightarrow{AB}| = \sqrt{3^2 + 2^2 + (-2)^2}$

$= \sqrt{9+4+4} = \boxed{\sqrt{17}}$

4. Let $\vec{u} = 2i + j + k$ and $\vec{v} = i + 4j - k$. Find $\vec{w} = \vec{u} - \vec{v}$ and find the unit vector $\vec{a}$ in the same direction as $\vec{w}$

$\vec{w} = \langle 2-1,\ 1-4,\ 1-1\rangle$

$= \boxed{\langle 1, -3, 0\rangle}$

$\vec{a} = \frac{1}{\sqrt{10}}\langle 1, -3, 0\rangle$

$\sqrt{1^2 + (-3)^2 + 0^2} = \sqrt{1+9}$

$= \sqrt{10}$

$= \boxed{\left\langle \frac{1}{\sqrt{10}},\ \frac{-3}{\sqrt{10}},\ 0\right\rangle}$