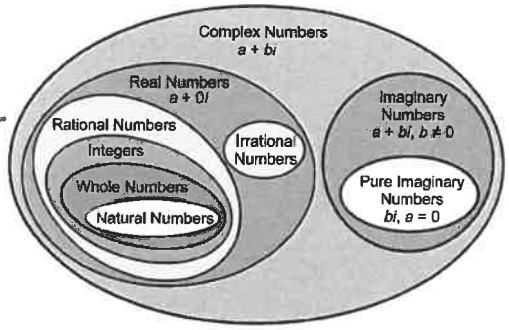
Can you solve $x^2 = -1$?

$x^2 = -1$

$x = \sqrt{-1}$

~~No solution~~

No "real" solution



Complex Number

$$i = \sqrt{-1}, \text{ so } i^2 = -1$$

If a and b are real numbers, then any number of the form

$$a + bi$$

is a complex number. In the complex number $a + bi$, a is the real part and b is the imaginary part

Two complex numbers $a + bi$ and $c + di$ are equal provided that their real parts are equal and their imaginary parts are equal → that is, they are equal if and only if $a = c$ and $b = d$.

so $3 + 2i = 3 + 2i$

$3 + 2i \neq 2 + 3i$

Standard Form: $a + bi$

$$\sqrt{-a} = i\sqrt{a}$$

$\sqrt{-a} = \sqrt{-1}\sqrt{a}$

$= i\sqrt{a}$

Caution:

First apply the definition $\sqrt{-a} = i\sqrt{a}$ before using any of the other rules for radicals.

1. Write each number as the product of a real number and i

   a. $\sqrt{-16}$

   $= i\sqrt{16}$

   $= 4i$

   b. $\sqrt{-70}$

   $= i\sqrt{70}$

2. Find each product or quotient. Simplify your answers.

   a. $\sqrt{-17} \cdot \sqrt{-17}$

   $= i\sqrt{17} \cdot i\sqrt{17}$

   $= i^2\sqrt{17^2}$

   $= -17$

   b. $\frac{\sqrt{-70}}{\sqrt{-7}}$

   $= \frac{i\sqrt{70}}{i\sqrt{7}}$

   $= \sqrt{\frac{70}{7}}$

   $= \sqrt{10}$

   c. $\frac{\sqrt{-12} \cdot \sqrt{-6}}{\sqrt{8}}$

   $= \frac{i\sqrt{12}\, i\sqrt{6}}{\sqrt{8}}$

   $= i^2\sqrt{\frac{72}{8}}$

   $= -\sqrt{9}$

   $= -3$

3. Write each number in standard form $a + bi$

a. $\frac{-9-\sqrt{-18}}{3}$

$= \frac{-9 - i\sqrt{18}}{3}$

$= \frac{-9 - 3i\sqrt{2}}{3}$

$= -3 - i\sqrt{2} = -3 - \sqrt{2}i$

b. $\frac{20+\sqrt{-8}}{2}$

$= \frac{20 + 2i\sqrt{2}}{2}$

$= 10 + i\sqrt{2}$

$= 10 + \sqrt{2}i$

4. Find each sum or difference. Write answers in standard form.

a. $(4 - i) + (8 + 5i)$

$= 12 + 4i$

b. $(-3 + 2i) - (-4 + 2i)$

$= -3 + 2i + 4 - 2i$

$= 1$

5. Find each product. Write answers in standard form.

* conjugates *

a. $(-2 + 3i)(4 - 2i)$

$= -8 + 12i + 4i - 6i^2$

$= -8 + 16i - 6(-1)$

$= -2 + 16i$

b. $(1 + 3i)(2 - 5i)$

$= 2 + 6i - 5i - 15i^2$

$= 2 + i + 15$

$= 17 + i$

c. $(2 + i)^2$

$= (2 + i)(2 + i)$

$= 4 + 4i + i^2$

$= 3 + 4i$

d. $(\sqrt{2} - 4i)(\sqrt{2} + 4i)$

$= 2 + 4i\sqrt{2} - 4i\sqrt{2} - 16i^2$

$= 2 + 16$

$= 18$

e. $(3 - i)(3 + i)(2 - 6i)$

$= (9 - i^2)(2 - 6i)$

$= 10(2 - 6i)$

$= 20 - 60i$

Property of Complex Conjugates
For real numbers a and b,

$$(a + bi)(a - bi) = a^2 + b^2$$

6. Find each quotient. Write answers in standard form.

a. $\frac{-5}{i}$

$= \frac{-5}{i} \cdot \frac{i}{i}$

$= \frac{-5i}{i^2}$

$= 5$

b. $\frac{-3+4i}{2-i} \cdot \frac{2+i}{2+i}$

$= \frac{-6+8i-3i+4i^2}{4-i^2}$

$= \frac{-10+5i}{5}$

$= -2+i$

c. $\frac{14+5i}{3+2i} \cdot \frac{3-2i}{3-2i}$

$= \frac{42+15i-28i-10i^2}{9-4i^2}$

$= \frac{52-13i}{13}$

$= 4-i$

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i = -\sqrt{-1}$
$i^4 = 1$

7. Simplify each power of i

a. i^{29}

$= i^5 i^5 i^5 i^5 i^5 i^4$

$= \sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1}\sqrt{-1}(1)$

$= \sqrt{-1}$

OR

$= i^4 i^4 i^4 i^4 i^4 i^4 i^4 i$

$= (1)(1)(1)(1)(1)(1)(1)\sqrt{-1}$

$= \sqrt{-1}$

b. i^{40}

$= \underbrace{i^4 \cdot i^4 \cdots i^4}_{10 \text{ times}}$

$= 1$

Homework:

Pg. 111
21, 29, 33, 39, 41, 43, 47, 49, 55, 59, 65, 73, 75, 83, 89

Most Difficult First:

Pg. 111
46, 51, 69, 77, 99