1.3 Challenge Problems
Honors Algebra 2 with Trig

1. NCTM May 2019 #7
For what values of $a$ will the roots of the equation $x^2 - 2x + (a^2 - 3) = 0$ be imaginary?

$b^2 - 4ac = \text{neg}$

$(-2)^2 - 4(1)(a^2-3)$

$4 - 4a^2 + 12$

$16 - 4a^2$

$16 - 4a^2 < 0$

$-4a^2 < -16$

$a^2 > 4$

$|a| > 2$

$a > 2$ or $a < -2$

$a < -2$ or $a > 2$

2. NCTM May 2019 #17
For what values of $a$ will the roots of the equation
$$(2a + 1)x^2 - (4a + 2)x + (2a - 1) = 0$$
be imaginary?

$b^2 - 4ac = \text{neg}$

$[-(4a+2)]^2 - 4(2a+1)(2a-1)$

$16a^2 + 16a + 4 - 4(4a^2 - 1)$

$16a^2 + 16a + 4 - 16a^2 + 4$

$16a + 8$

$16a + 8 < 0$

$a < -1/2$

3. NCTM Sept 2018 #8
Find two numbers $a$ and $b$ such that $a = b^2$ and $b = a^2$ but $a \neq b$.

$a = (a^2)^2$

$a = a^4$

$a^4 - a = 0$

$a(a^3 - 1) = 0$

$a(a-1)(a^2 + a + 1) = 0$

$a = 0 \quad a = 1$
↖ would have $a = b$

$$a = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

4. NCTM Sept 2018 #3
Given that $i = \sqrt{-1}$, solve for $x$:

$$(x + i)(x - i) = 10$$

$x^2 - i^2 = 10$

$x^2 + 1 = 10$

$x^2 = 9$

$\boxed{x = \pm 3}$

5. NCTM Sept 2018 #18
There are 3 complex numbers that one can cube to get the number 8. Find all 3.

$x^3 = 8$

$x^3 - 8 = 0$

$(x-2)(x^2 + 2x + 4) = 0$

$x = 2$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2}$

$= \frac{-2 \pm \sqrt{4 - 16}}{2}$

$= \frac{-2 \pm \sqrt{-12}}{2}$

$= \frac{-2 \pm 2i\sqrt{3}}{2}$

$= -1 \pm i\sqrt{3}$

$\boxed{x = 2, -1 \pm i\sqrt{3}}$

6. NCTM May 2016 #15
Let $f(x) = x^2$ where $x = a + bi$, $i = \sqrt{-1}$, and $a$ and $b$ are real numbers. Compute $f(3 + 4i)$

$f(3+4i) = (3 + 4i)^2$

$= 9 + 24i + 16i^2$

$= 9 + 24i - 16$

$= -7 + 24i$