Student Objectives:

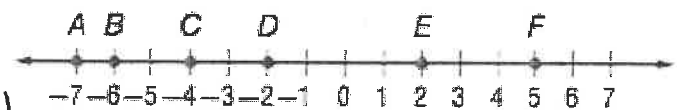
- The student will be able to calculate distance and midpoint between two points.
- The student will be able to apply the definition of a midpoint to calculate length of a segment.
- The student will be able to recall definitions of angle, circle, perpendicular line, parallel lines, and line segment, based on understanding definitions of point, line, distance along a line, and distance around a circular arc.
- The student will be able to make formal geometric construction with a variety of tools and methods.

Distance: length of a segment between endpoints

line segment → two points as endpoints and all collinear points in between

ex) A B $\overline{AB}$ or $\overline{BA}$

1. Use the number line to find:



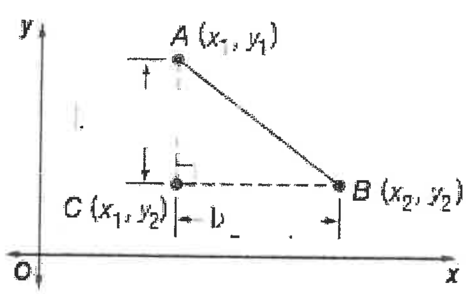
a. $BE = |x_2 - x_1|$
$= |2 - (-6)|$
$= 8$

b. $AC =$ 3

c. $CF =$ 9

d. $FB =$ 11

Prove distance formula



$(CB)^2 + (AC)^2 = (AB)^2$

$(|x_2 - x_1|)^2 + (|y_2 - y_1|)^2 = (AB)^2$

$(x_2 - x_1)^2 + (y_2 - y_1)^2 = (AB)^2$

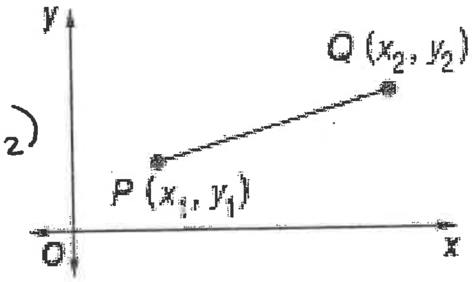
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = AB$

* square always pos so don't need absolute value

Distance Formula:

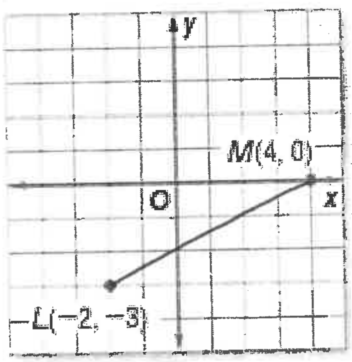
If P has coordinates (x_1, y_1)
and Q has coordinates (x_2, y_2)
then

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



2. Find the distance between each pair of points:

a.



$$LM = \sqrt{(4 - (-2))^2 + (0 - (-3))^2}$$

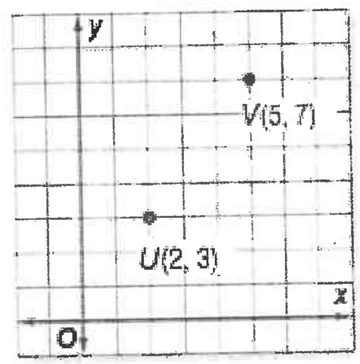
$$= \sqrt{6^2 + 3^2}$$

$$= \sqrt{36 + 9}$$

$$= \sqrt{45}$$

$$\approx 6.7 \text{ units}$$

b.



$$UV = \sqrt{(2-5)^2 + (3-7)^2}$$

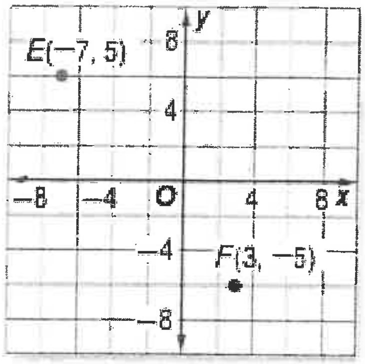
$$= \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

c.

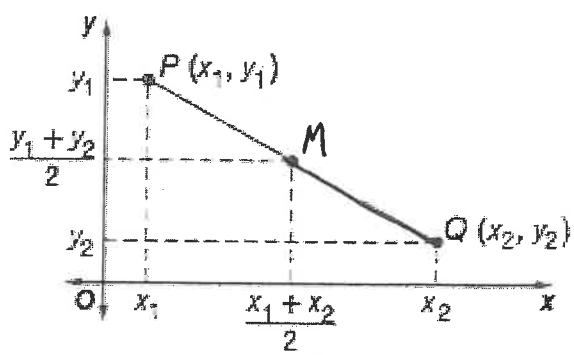


$$EF = \sqrt{(-7-3)^2 + (5-(-5))^2}$$
$$= \sqrt{(-10)^2 + (10)^2}$$
$$= \sqrt{200}$$
$$\approx 14.1 \text{ units}$$

Midpoint: of a segment is the point halfway between the endpoints of the segment

The Midpoint of a line segment is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



3. Find the coordinates of the midpoint of a segment with the given endpoints:

a. $W(12,2)$, $X(7,9)$

$x_1\ y_1 \quad x_2\ y_2$

$$M = \left(\frac{12+7}{2}, \frac{2+9}{2}\right)$$

$$= \left(\frac{19}{2}, \frac{11}{2}\right)$$

b. $V(-2,5)$, $Z(3,-17)$

$x_1\ y_1 \quad x_2\ y_2$

$$M = \left(\frac{-2+3}{2}, \frac{5+(-17)}{2}\right)$$

$$= \left(\frac{1}{2}, \frac{-12}{2}\right)$$

$$= \left(\frac{1}{2}, -6\right)$$

4. Find the coordinate of the missing endpoint if B is the midpoint of $\overline{AC}$.

a. $A(1,7)$, $B(-3,1)$

$x_1\ y_1 \quad x_M\ y_M$

midpoint

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (x_M, y_M)$$

$$x_M = \frac{x_1+x_2}{2}$$

$$-3 = \frac{1+x_2}{2}$$

$$-6 = 1 + x_2$$

$$-7 = x_2$$

$$y_M = \frac{y_1+y_2}{2}$$

$$1 = \frac{7+y_2}{2}$$

$$2 = 7 + y_2$$

$$-5 = y_2$$

$$\boxed{C(-7,-5)}$$

b. $C(-6,-2)$, $B(-3,-5)$

$x_1\ y_1 \quad x_M\ y_M$

$$x_M = \frac{x_1+x_2}{2} \qquad y_M = \frac{y_1+y_2}{2}$$

$$-3 = \frac{-6+x_2}{2} \qquad -5 = \frac{-2+y_2}{2}$$

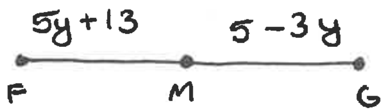
$$-6 = -6 + x_2 \qquad -10 = -2 + y_2$$

$$0 = x_2 \qquad -8 = y_2$$

$$\boxed{C(0,-8)}$$

5. Suppose M is the midpoint of $\overline{FG}$. Use the given information to find the missing measure or value.

a. $FM = 5y + 13$,
$MG = 5 - 3y$, $FG = ?$



$5y + 13 = 5 - 3y$

$8y = -8$

$y = -1$

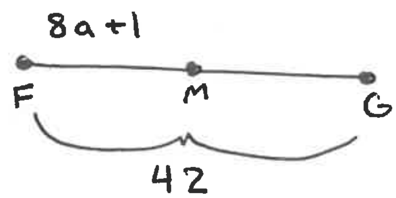
$FG = 5y + 13 + 5 - 3y$

$= 2y + 18$

$= 2(-1) + 18$

$\boxed{= 16}$

b. $FM = 8a + 1$, $FG = 42$,
$a = ?$



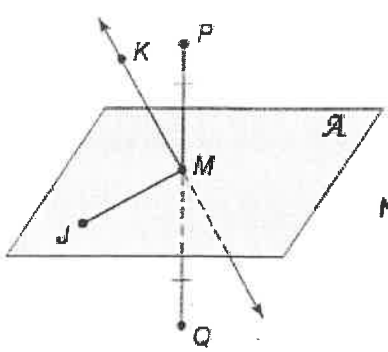
$42 = 2(8a + 1)$

$42 = 16a + 2$

$40 = 16a$

$\boxed{2.5 = a}$

Segment Bisector: any segment, line, or plane that intersects a segment at its midpoint

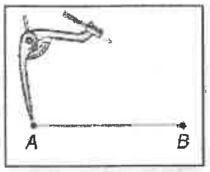


M is the midpoint of $\overline{PQ}$

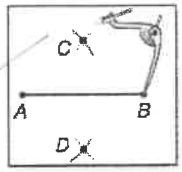
$\Rightarrow$ plane A, $\overline{MJ}$ and $\overleftrightarrow{KM}$ are all bisectors of $\overline{PQ}$

Construction Bisect a Segment

Step 1 Draw a segment and name it $\overline{AB}$. Place the compass at point A. Adjust the compass so that its width is greater than $\frac{1}{2}AB$. Draw arcs above and below $\overline{AB}$.



Step 2 Using the same compass setting, place the compass at point B and draw arcs above and below $\overline{AB}$ so that they intersect the two arcs previously drawn. Label the points of the intersection of the arcs as C and D.



Step 3 Use a straightedge to draw $\overline{CD}$. Label the point where it intersects $\overline{AB}$ as M. Point M is the midpoint of $\overline{AB}$, and $\overline{CD}$ is a bisector of $\overline{AB}$.