Dot Product

The geometric definition of the dot product of two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ is the number:

$$\mathbf{a} \cdot \mathbf{b} = |a||b| \cos\theta$$

where θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, $0 \leq \theta \leq \pi$. If either $\mathbf{a}$ or $\mathbf{b}$ is **0**, then $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$

The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (perpendicular) if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

i.e. the angle between them is _____

2 Properties of the Dot Product If **a**, **b**, and **c** are vectors in V_3 and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $\mathbf{0} \cdot \mathbf{a} = 0$

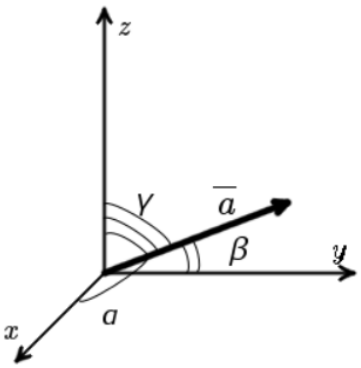
1. Find the following using the vectors $\mathbf{a} = \langle -1, -2, -3 \rangle$, $\mathbf{b} = \langle -10, 2, 1 \rangle$, and $\mathbf{c} = \langle 2, 8, -6 \rangle$
   a. $\mathbf{a} \cdot \mathbf{b}$
   b. $\mathbf{a} \cdot \mathbf{c}$
   c. Find the angle between $\mathbf{a}$ and $\mathbf{b}$

2. If $|\mathbf{a}| = 1$ and $|\mathbf{b}| = 2$, what is the maximum for $\mathbf{a} \cdot \mathbf{b}$? What does this say about the vectors?

Direction Cosines:

The direction cosines of vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ can be found using:

$$\cos\alpha = \frac{a_1}{|a|}; \qquad \cos\beta = \frac{a_2}{|a|}; \qquad \cos\gamma = \frac{a_3}{|a|}$$



3. Find the direction angles for $\mathbf{a} = \langle 1, 0, 5 \rangle$

Projections

Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: $comp_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of $\mathbf{b}$ onto $\mathbf{b}$: $proj_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

4. Find the vector and scalar projections of $\mathbf{m} = \langle 2, 1, 5 \rangle$ onto $\mathbf{n}\langle 1, 2, 3, \rangle$