Dot Product

The geometric definition of the dot product of two nonzero vectors a and b is the number:

$$\mathbf{a} \cdot \mathbf{b} = |a||b| \cos \theta$$

where θ is the angle between the vectors a and b, $0 \leq \theta \leq \pi$. If either a or b is **0**, then $\mathbf{a} \cdot \mathbf{b} = 0$

The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $(b) = \langle b_1, b_2, b_3 \rangle$ is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Two non-zero vectors a and b are orthogonal (perpendicular) if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

i.e. the angle between them is $\pi/2$

$\boxed{2}$ **Properties of the Dot Product** If **a**, **b**, and **c** are vectors in V_3 and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $\mathbf{0} \cdot \mathbf{a} = 0$

1. Find the following using the vectors $\mathbf{a} = \langle -1, -2, -3 \rangle$, $\mathbf{b} = \langle -10, 2, 1 \rangle$, and $\mathbf{c} = \langle 2, 8, -6 \rangle$

a. $\mathbf{a} \cdot \mathbf{b}$

$a \cdot b = (-1)(-10) + (-2)(2) + (-3)(1)$

$= 10 - 4 - 3$

$= 3$

b. $\mathbf{a} \cdot \mathbf{c}$

$a \cdot c = (-1)(2) + (-2)(8) + (-3)(-6)$

$= -2 - 16 + 18$

$= 0$

vectors $\vec{a}$ & $\vec{c}$ are $\perp$

c. Find the angle between a and b

$a \cdot b = |a|\,|b| \cos \theta$

$3 = \sqrt{14}\sqrt{105} \cos \theta$

$\cos^{-1}\left(\frac{3}{\sqrt{14}\sqrt{15}}\right) = \theta$

$\theta = 1.493 \text{ rad} = 85.51^\circ$

$|a| = \sqrt{(-1)^2 + (-2)^2 + (-3)^2}$

$= \sqrt{14}$

$|b| = \sqrt{(-10)^2 + 2^2 + 1^2}$

$= \sqrt{105}$

2. If $|\mathbf{a}| = 1$ and $|\mathbf{b}| = 2$, what is the maximum for $\mathbf{a} \cdot \mathbf{b}$? What does this say about the vectors?

$a \cdot b = |a|\,|b| \cos\theta$

$= 1 \cdot 2 \cos\theta$

$\theta = 0$ so $\cos\theta = 1$

max for $a \cdot b = 2$

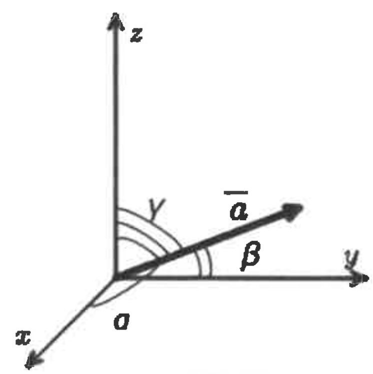
$\vec{a}$ and $\vec{b}$ point in same direction

* angle in between them is 0

Direction Cosines:

The direction cosines of vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ can be found using:

$\cos\alpha = \frac{a_1}{|a|};$ $\quad \cos\beta = \frac{a_2}{|a|};$ $\quad \cos\gamma = \frac{a_3}{|a|}$



3. Find the direction angles for $\mathbf{a} = \langle 1, 0, 5 \rangle$

$\cos\alpha = \frac{1}{\sqrt{26}}$ $\qquad \alpha = \cos^{-1}(1/\sqrt{26}) = 78.7°$

$\cos\beta = 0/\sqrt{26}$ $\qquad \beta = 90°$

$\cos\gamma = 5/\sqrt{26}$ $\qquad \gamma = \cos^{-1}(5/\sqrt{26}) = 11.3°$

$|a| = \sqrt{1 + 0 + 25}$

$= \sqrt{26}$

Projections

Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: $comp_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of $\mathbf{b}$ onto $\mathbf{b}$: $proj_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

4. Find the vector and scalar projections of $\mathbf{m} = \langle 2, 1, 5 \rangle$ onto $\mathbf{n}\langle 1, 2, 3, \rangle$

$|n| = \sqrt{1 + 4 + 9}$

$= \sqrt{14}$

$comp_n m = \frac{m \cdot n}{|n|} = \frac{(2)(1) + (1)(2) + (5)(3)}{\sqrt{14}}$

$= 19/\sqrt{14}$

$proj_n m = \frac{19}{\sqrt{14}^2} \langle 1, 2, 3 \rangle = \langle 19/14, 38/14, 57/14 \rangle$