Reviewing the Determinate

The determinate of a 2x2 matrix is computed by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The determinate of a 3x3 matrix is computed by

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Example: Find the determinate of this matrice.

$$\begin{vmatrix} 1 & 3 & 4 \\ 5 & 0 & 2 \\ -3 & 6 & 7 \end{vmatrix}$$

Properties of the Cross Product: If **a**, **b**, and **c** are vectors and d is a scalar, then

- $\mathbf{a} \times \mathbf{a} = \mathbf{0}$
- $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a}) = -\mathbf{b} \times \mathbf{a}$
- $(d\mathbf{a}) \times \mathbf{b} = d(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (d\mathbf{b})$
- $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
- $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then $\mathbf{a} \times \mathbf{b} =$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

1. Compute the following for the vectors $\mathbf{a} = \langle 1, 3, 4 \rangle$ and $\mathbf{b} = \langle 2, -5, 6 \rangle$.

    a. $\mathbf{a} \times \mathbf{b}$

    b. $\mathbf{b} \times \mathbf{a}$

    c. $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$

2. Find a vector orthogonal to the plane determined by the points $A(1, 2, 3)$, $B(4, 6, 8)$ and $C(15, 2, - 5)$

Theorem: The magnitude of $\mathbf{A} \times \mathbf{B}$ is

$$|\mathbf{A} \times \mathbf{B}| \quad = \quad |\mathbf{A}||\mathbf{B}| \sin \theta, \text{ where } \theta \text{ is the angle between them}$$

$$= \quad \text{area of the parallelogram spanned by } \mathbf{A} \text{ and } \mathbf{B}.$$

3. The points $P(1, 2, 1)$, $Q(1, 0, 0)$ and $R(0, 3, 1)$ create a triangle. What is the area of the triangle?