Quadratic Equation in One Variable
An equation can be written in the form
$$ax^2 + bx + c = 0$$
where a, b, and c are real numbers with $a \neq 0$, is a quadratic equation. The given form is called standard form.

Quadratic equation is a 2nd Degree equation.

Zero-Factor Property
If a and b are complex numbers with $ab = 0$, then $a = 0$ or $b = 0$ or both equal zero.

1. Solve each equation using the zero-factor property.
   a. $x^2 + 2x - 8 = 0$
   $(x+4)(x-2)=0$
   $\boxed{x=-4,2}$
   b. $-6x^2 + 7x = -10$
   $-6x^2+7x+10=0$
   $6x^2-7x-10=0$
   $(x-2)(6x+5)=0$
   $\boxed{x=-5/6, 2}$

Square Root Property
If $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$

2. Solve each equation using the square root property.
   a. $x^2 = 121$
   $\boxed{x=\pm 11}$
   b. $48 - x^2 = 0$
   $48=x^2$
   $\pm\sqrt{48}=x$
   $\boxed{\pm 4\sqrt{3}=x}$
   c. $(4x+1)^2 = 20$
   $4x+1=\pm\sqrt{20}$
   $4x+1=\pm 2\sqrt{5}$
   $4x=-1\pm 2\sqrt{5}$
   $\boxed{x=\dfrac{-1\pm 2\sqrt{5}}{4}}$
   d. $(x-4)^2=-5$
   $x-4=\pm\sqrt{-5}$
   $\boxed{x=4\pm\sqrt{5}\,i}$
   e. $(-2x+5)^2 = -8$
   $-2x+5=\pm\sqrt{-8}$
   $-2x+5=\pm 2i\sqrt{2}$
   $-2x=-5\pm 2\sqrt{2}\,i$
   ★ Not in standard form
   $\boxed{x=\dfrac{5\pm 2\sqrt{2}\,i}{2}}$

$\frac{5-2}{2}$ 3/2 $\frac{5+2}{2}$ 7/2 $\frac{-5+2}{-2}$ 3/2 $\frac{-5-2}{-2}$ 7/2

Steps to **Completing the Square**

$$ax^2 + bx + c = 0,\ a \neq 0$$

I. If $a \neq 1$ divide each side of the equation by a

II. Rewrite the equation so that the constant term is alone on one side of the equality symbol

III. Square half the coefficient of x

  A. Add this square to each side of the equation

IV. Factor the resulting trinomial as a perfect square

V. Use the square root property to complete the solution

3. Solve each equation using completing the square.

a. $x^2 - 7x + 12 = 0$

$x^2 - 7x = -12$

$x^2 - 7x + (7/2)^2 = -12 + (7/2)^2$

$(x - 7/2)^2 = -12 + 49/4$

$x - 7/2 = \pm\sqrt{-\frac{48}{4} + \frac{49}{4}}$

$x = 7/2 \pm \sqrt{1/4}$

$x = 7/2 \pm 1/2$ → $x = 8/2,\ 6/2$

$\boxed{x = 4,\ 3}$

c. $x^2 - 10x + 18 = 0$

$x^2 - 10x = -18$

$x^2 - 10x + 25 = -18 + 25$

$(x - 5)^2 = 7$

$\boxed{x = 5 \pm \sqrt{7}}$

b. $4x^2 - 3x - 10 = 0$

$x^2 - 3/4x - 10/4 = 0$

$x^2 - 3/4x = 5/2$

$x^2 - 3/4x + (3/8)^2 = 5/2 + (3/8)^2$

$(x - 3/8)^2 = 5/2 + 9/64$

$x - 3/8 = \pm\sqrt{160/64 + 9/64}$

$x = 3/8 \pm \sqrt{169/64}$

$x = 3/8 \pm 13/8$

$\boxed{x = 2,\ 5/4}$

d. $3x^2 + 2x = 5$

$x^2 + 2/3x = 5/3$

$x^2 + 2/3x + (1/3)^2 = 5/3 + (1/3)^2$

$(x + 1/3)^2 = \frac{5}{3} + \frac{1}{9}$

$x + 1/3 = \pm\sqrt{\frac{15}{9} + \frac{1}{9}}$

$x = -\frac{1}{3} \pm \sqrt{16/9}$

$x = -1/3 \pm 4/3$

$x = 3/3,\ -5/3$

$\boxed{x = 1,\ -5/3}$

$32 \times 5 = 160$

Quadratic Formula:

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Solve each equation using the quadratic formula.

a. $x^2 - 3x - 2 = 0$

$x = \frac{3 \pm \sqrt{9 - 4(1)(-2)}}{2(1)}$

$= \frac{3 \pm \sqrt{9 + 8}}{2}$

$= \boxed{\frac{3 \pm \sqrt{17}}{2}}$

b. $x^2 - 4x = -1$

$x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$

$= \frac{4 \pm \sqrt{16 - 4}}{2}$

$= \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm 2\sqrt{3}}{2} = \boxed{2 \pm \sqrt{3}}$

c. $-6x^2 = 3x + 2$

$0 = 6x^2 + 3x + 2$

$x = \frac{-3 \pm \sqrt{9 - 4(6)(2)}}{2(6)}$

$= \frac{-3 \pm \sqrt{9 - 48}}{12}$

$= \frac{-3 \pm \sqrt{-39}}{12}$

$= \boxed{\frac{-3 \pm \sqrt{39}\, i}{12}}$

5. Solve each cubic using factoring and the quadratic formula

a. $x^3 - 27 = 0$

$(x - 3)(x^2 + 3x + 9) = 0$

$\boxed{x = 3}$

$x = \frac{-3 \pm \sqrt{9 - 4(1)(9)}}{2(1)}$

$= \frac{-3 \pm \sqrt{9 - 36}}{2}$

$= \frac{-3 \pm \sqrt{-27}}{2}$

$\boxed{x = \frac{-3 \pm 3\sqrt{3}\, i}{2}}$

b. $x^3 + 64 = 0$

$(x + 4)(x^2 - 4x + 16) = 0$

$\boxed{x = -4}$

$x = \frac{4 \pm \sqrt{16 - 4(1)(16)}}{2(1)}$

$= \frac{4 \pm \sqrt{16 - 64}}{2}$

$= \frac{4 \pm \sqrt{-48}}{2}$

$= \frac{4 \pm 4\sqrt{3}\, i}{2}$

$= \boxed{2 \pm 2\sqrt{3}\, i}$

6. Solve each equation for the specified variable.

a. $E = \frac{e^2 k}{2r}$, for e

$2rE = e^2 k$

$\frac{2rE}{k} = e^2$

$\pm\sqrt{\frac{2rE}{k}} = e$

* rationalize

$\pm\frac{\sqrt{2rE}}{\sqrt{k}} \cdot \frac{\sqrt{k}}{\sqrt{k}} = e$

$\boxed{\pm\frac{\sqrt{2rEk}}{k} = e}$

Discriminant:

Solutions of Quadratic Equations when a, b, and c are integers

Discriminant	Number of Solutions	Type of Solutions
Positive, perfect square	Two	Rational
Positive, but not a perfect square	Two	Irrational
Zero	One (double solution)	Rational
Negative	Two	Nonreal Complex

7. Evaluate the discriminant for each equation. Then use it to determine the number of distinct solutions, and tell whether they are rational, irrational, or nonreal complex numbers. (Do not solve the equation).

a. $x^2 + 4x + 4 = 0$

$\sqrt{4^2 - 4(1)(4)}$

$= \sqrt{16 - 16}$

$= 0$

$\boxed{\text{One, Rational}}$

b. $8x^2 = -14x - 3$

$0 = 8x^2 + 14x + 3$

$\sqrt{14^2 - 4(8)(3)}$

$= \sqrt{196 - 96}$

$= \sqrt{100}$

$= 10$

$\boxed{\text{Two, Rational}}$

c. $2x^2 + 4x + 1 = 0$

$\sqrt{4^2 - 4(2)(1)}$

$= \sqrt{16 - 8}$

$\sqrt{8}$

$\boxed{\text{Two, Irrational}}$

8. A piece of machinery produces rectangular sheets of metal such that the length is three times the width. Equal-sized squares measuring 5 in. on a side can be cut from the corners so that the resulting piece of metal can be shaped into an open box by folding up the flaps. If specifications call for the volume of the box to be 1435 in^3, find the dimensions of the original piece of metal.

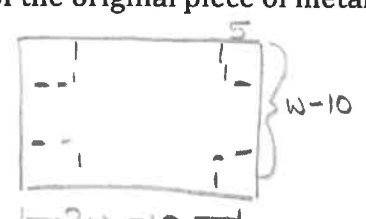
$l = 3w$

$V = 1435$

$V = lwh$

$l = 3w - 10 \quad h = 5$

$w = w - 10$



$V = (3w-10)(w-10)(5)$

$1435 = 15w^2 - 200w + 500$

$0 = 15w^2 - 200w - 935$

$0 = 3w^2 - 40w - 187$

$0 = (3w+11)(w-17)$

$\cancel{w = -11/3} \quad w = 17$

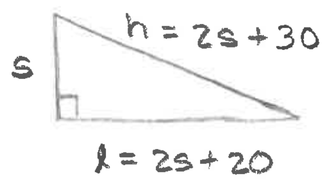
$l = 3(17)$

$= 51$

$\boxed{\text{original dimensions would be } 17\text{in} \times 51\text{in}}$

9. A piece of property has the shape of a right triangle. THe longer leg is 20 m longer than twice the length of the shorter leg. The hypotenuse is 10 m longer than the length of the longer leg. Find the lengths of the sides of the triangular lot.

Back↑



$$s^2 + l^2 = h^2$$

$$s^2 + (2s+20)^2 = (2s+30)^2$$

$$s^2 + 4s^2 + 80s + 400 = 4s^2 + 120s + 900$$

$$s^2 - 40s - 500 = 0$$

$$(s-50)(s+10) = 0$$

$$\cancel{s = -10} \quad s = 50$$

$$\boxed{\begin{array}{l} s = 50\text{ m} \\ l = 120\text{m} \\ h = 130\text{ m} \end{array}}$$

10. If a projectile is launched vertically upward from the ground with an initial velocity of 100 ft per sec, neglecting air resistance, its height s (in feet) above the ground t seconds after projection is given by

$$s = -16t^2 + 100t$$

a. After how many seconds will it be 50 ft above the ground?

$s = 50$

$$50 = -16t^2 + 100t$$

$$16t^2 - 100t + 50 = 0$$

$$8t^2 - 50t + 25 = 0$$

$$t = \frac{50 \pm \sqrt{(50)^2 - 4(8)(25)}}{2(8)} \rightarrow t = \frac{50 \pm \sqrt{1700}}{16}$$

$$\boxed{t \approx 0.55 \text{ or } 5.70}$$

b. How long will it take for the projectile to return to the ground?

$s = 0$

$$0 = -16t^2 + 100t$$

$$0 = -4t(4t - 25)$$

$$t = 0, \ 25/4$$

$$\boxed{t = 25/4 = 6.25\text{ sec}}$$

Homework:

Pg. 121
15, 21, 25, 27, 33, 35, 39, 41, 43, 53, 67, 73, 83, 85

Pg. 131
27, 35, 47

Most Difficult First:

Pg. 121
35, 48, 64, 78, 91

Pg. 130
24, 36