Equations of a Line (Point-Direction Form)

The line L through the point $P_0 = (x_0, y_0, z_0)$ in the direction of $\mathbf{v} = \langle a, b, c \rangle$ is described by

Vector Parametrization:

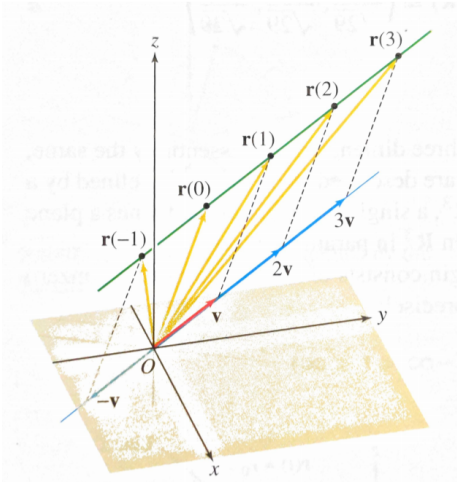
$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v} = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

where $\mathbf{r}_0 = \overrightarrow{0P_0}$.

Parametric Equations:

$$x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct$$

The parameter t takes on values $-\infty < t < \infty$



DF FIGURE 13 The terminal point of $\mathbf{r}(t)$ traces out a line as t varies from $-\infty$ to ∞.

1. Find the vector equation and the parametric equations of a line through the point (1, 2, 3) where the line is parallel to the vector $\mathbf{v} = \langle 2, 5, 10 \rangle$.

2. Find the vector equation of the line through the points (3, 5, 5) and (2, 1 − 5). Also give the parametric equations of this line. Where does the line intersect the xy-plane?

3. Is the point (7, 10, 17) on the line $\mathbf{r} = \langle 1 + 3t, 2 + 4t, 3 + 7t \rangle$?

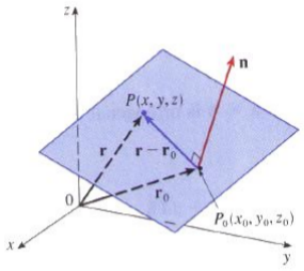
Plane is determined by a point $P_0(x_0, y_0, z_0)$ and a vector $\mathbf{n} = \langle a, b, c \rangle$ that is orthogonal to the plane. The vector $\mathbf{n}$ is called a normal vector.

Vector Form: $\mathbf{n} \cdot \langle x, y, z \rangle = d$

Scalar Form:

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$ax + by + cz = d$



4. Find an equation of the plane through the point (1, 2, 3) and is orthogonal to the vector $\langle 3, 4, 7 \rangle$

5. Find an equation of the plane through the points $A(1, 1, 3)$, $B(-1, 3, 2)$ and $C(1, -1, 2)$.

6. Find an equation of the plane through the point (1, 2, 3) and contains the line $x = 2 + 4t,\ y = 1 + 5t,\ z = -1 + 3t$.

Definition: Two planes are parallel if their normal vectors are parallel.

Definition: Two planes are perpendicular(orthogonal) if their normal vectors are perpendicular.

Definition: The angle between two non-parallel planes is the acute angle between the normal vectors.

7. Determine if there are pairs of planes (listed below) that are parallel, orthogonal, or neither?

   P_1: $4x + 2y - 8x = 15$

   P_2: $2x + y - 4z = 12$

   P_3: $3x + 2y + 2z = 10$