Rational Equation: an equation that has a rational expression for one or more terms.

*When solving rational equations always FACTOR first. Then multiple by the LCD

1. Solve each equation. Be sure to check for extraneous solutions!

a. $\frac{3x-1}{3} - \frac{2x}{x-1} = x$

LCD → $3(x-1)$

$$3(x-1) \cdot \left[\frac{3x-1}{3} - \frac{2x}{x-1} = \frac{x}{1}\right]$$

$$\frac{3(x-1)(3x-1)}{3} - \frac{3(x-1)(2x)}{x-1} = 3(x-1)x$$

$$(x-1)(3x-1) - 3(2x) = 3x(x-1)$$

$$3x^2-3x-x+1-6x = 3x^2-3x$$

$$-10x+1 = -3x$$

$$1 = 7x$$

$$\boxed{1/7 = x}$$

Check

$$\frac{3(1/7)-1}{3} - \frac{2(1/7)}{1/7-1} \stackrel{?}{=} 1/7$$

True ✓

b. $\frac{x}{x-2} = \frac{2}{x-2} + 2$

LCD → $x-2$

$$(x-2)\left[\frac{x}{x-2} = \frac{2}{x-2} + 2\right]$$

$$\frac{x(x-2)}{x-2} = \frac{2(x-2)}{x-2} + 2(x-2)$$

$$x = 2 + 2x - 4$$

$$x = 2x - 2$$

$$x = 2$$

Check

$x = 2$ makes den. 0

$\boxed{\text{No solution}}$

c. $\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x^2-2x}$

$$\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x(x-2)}$$

LCD → $x(x-2)$

$$x(x-2)\left[\frac{3x+2}{x-2} + \frac{1}{x} = \frac{-2}{x(x-2)}\right]$$

$$\frac{x(x-2)(3x+2)}{x-2} + \frac{x(x-2)}{x} = \frac{-2x(x-2)}{x(x-2)}$$

$$x(3x+2) + (x-2) = -2$$

$$3x^2+2x+x-2 = -2$$

$$3x^2+3x-2 = -2$$

$$3x^2+3x = 0$$

$$3x(x+1) = 0$$

$$x = 0, -1 \qquad \boxed{x=-1}$$

Check

$x=0$ make den 0

$$\frac{3(-1)+2}{-1+2} + \frac{1}{-1} = \frac{-2}{(-1)(-1-2)}$$

True

d. $\frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{x^2-1}$

$$\frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{(x+1)(x-1)}$$

$$(x+1)(x-1)\left[\frac{-4x}{x-1} + \frac{4}{x+1} = \frac{-8}{(x+1)(x-1)}\right]$$

$$-4x(x+1) + 4(x-1) = -8$$

$$-4x^2 - 4x + 4x - 4 = -8$$

$$4x^2 - 4 = 0$$

$$4(x-1)(x+1) = 0$$

$$x = \pm 1$$

Check

both $x=1$ and $x=-1$ make den $= 0$

$\boxed{\text{No solution}}$

Solving an Equation Involving Radicals

Step 1: Isolate the radical on one side

Step 2: Raise each side of the equation to a power that is the same as the index of the radical so that the radical is eliminated

If the equation still contains a radical, repeat Steps 1 and 2.

Step 3: Solve the resulting equation

Step 4: Check each solution in the original equation.

2. Solve each equation. Be sure to check for extraneous solutions!

a. $x - \sqrt{15 - 2x} = 0$

$$x = \sqrt{15-2x}$$

$$(x)^2 = \left(\sqrt{15-2x}\right)^2$$

$$x^2 = 15 - 2x$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$$x = -5, 3$$

$$\boxed{x = 3}$$

Check

$-5 - \sqrt{15-2(-5)} = 0$

$-5 - \sqrt{25} = 0$ ✗

False

$3 - \sqrt{15-2(3)} = 0$

$3 - \sqrt{9} = 0$ ✓

True

b. $\sqrt{2x+3} - \sqrt{x+1} = 1$

$\sqrt{2x+3} = 1 + \sqrt{x+1}$

$(\sqrt{2x+3})^2 = (1+\sqrt{x+1})^2$

$2x+3 = 1 + 2\sqrt{x+1} + (x+1)$

$2x+3 = 2 + x + 2\sqrt{x+1}$

* isolate again

$x + 1 = 2\sqrt{x+1}$

$\frac{1}{2}x + \frac{1}{2} = \sqrt{x+1}$

$(\frac{1}{2}x + \frac{1}{2})^2 = (\sqrt{x+1})^2$

$\frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4} = x+1$

$\frac{1}{4}x^2 - \frac{1}{2}x - \frac{3}{4} = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\boxed{x = 3, -1}$

Check

$\sqrt{2(3)+3} - \sqrt{3+1} = 1$

$\sqrt{9} - \sqrt{4} = 1$ ✓

True

$\sqrt{2(-1)+3} - \sqrt{-1+1} = 1$

$\sqrt{1} - 0 = 1$ ✓

True

c. $\sqrt[3]{4x^2 - 4x + 1} - \sqrt[3]{x} = 0$

$\sqrt[3]{4x^2-4x+1} = \sqrt[3]{x}$

$(\sqrt[3]{4x^2-4x+1})^3 = (\sqrt[3]{x})^3$

$4x^2 - 4x + 1 = x$

$4x^2 - 5x + 1 = 0$

$(4x-1)(x-1) = 0$

$\boxed{x = \frac{1}{4}, 1}$

Check

$\sqrt[3]{4(\frac{1}{4})^2 - 4(\frac{1}{4}) + 1} - \sqrt[3]{\frac{1}{4}} = 0$

$\sqrt[3]{\frac{1}{4} - 1 + 1} - \sqrt[3]{\frac{1}{4}} = 0$ ✓

True

$\sqrt[3]{4 - 4 + 1} - \sqrt[3]{1} = 0$ ✓

True

3. Solve each equation. Be sure to check your solutions.

a. $x^{\frac{3}{5}} = 27$

$\left(x^{3/5}\right)^{5/3} = (27)^{5/3}$

$x = \sqrt[3]{27}^{\,5}$

$x = 3^5$

$\boxed{x = 243}$

b. $(x-4)^{\frac{2}{3}} = 16$

$\left((x-4)^{2/3}\right)^{3/2} = 16^{3/2}$

$x - 4 = \pm\sqrt{16}^{\,3}$

$x - 4 = \pm 4^3$

$x - 4 = \pm 64$

$x = 4 \pm 64$

$\boxed{x = -60, 68}$

Check

$\sqrt[3]{(-60-4)}^{\,2} = 16$

$\sqrt[3]{-64}^{\,2} = 16$ ✓

True

$\sqrt[3]{68-4}^{\,2} = 16$

$\sqrt[3]{64}^{\,2} = 16$ ✓

True

4. Simplify $\sqrt{32x} + \sqrt{2x} - \sqrt{18x}$

$= 4\sqrt{2x} + \sqrt{2x} - 3\sqrt{2x}$

$= \boxed{2\sqrt{2x}}$

Most Difficult First:
Pg. 146:
#24, 58, 82