1.6 Day 2 Challenge Problems
Honors Algebra 2 with Trig

1. NCTM Dec/Jan 2014/15 #11

Given

$$f(x) = x+3$$
$$g(x) = ax^2+bx+c$$
$$g(f(x)) = 2x^2+7x+6,$$

find $a+b+c$.

$g(f(x)) = a(x+3)^2 + b(x+3) + c$

$= a(x^2+6x+9) + bx + 3b + c$

$= ax^2 + a6x + a9 + bx + 3b + c$

$2x^2 + 7x + 6 = ax^2 + a6x + a9 + bx + 3b + c$

$2x^2 + 7x + 6 = ax^2 + \underbrace{a6x + bx}_{x(a6+b)} + a9 + 3b + c$

$2 = a$

$7 = 6a + b$

$7 = 6(2) + b$

$b = -5$

$6 = 9a + 3b + c$

$6 = 9(2) + 3(-5) + c$

$3 = c$

$a+b+c = 2-5+3$

$= \boxed{0}$

2. NCTM March 2019
(There are two ways to solve this problem and maybe more than two. One way to solve this requires the given information that x is a prime integer less than 1000. The second way does not require that given information.)

Solve for x:

$$\sqrt{x+8} + \sqrt{x-32} = 20.$$

① x = prime integer

$x+8$ and $x-32$ must be perfect squares to equal 20

Try $x = 41$

$\sqrt{41+8} + \sqrt{41-32}$

$\sqrt{49} + \sqrt{9}$

$7 + 3$

10

Fails

Try $\boxed{x = 113}$

$\sqrt{113+8} + \sqrt{113-32}$

$11 + 9$

20 ✓

② $\sqrt{x-32} = 20 - \sqrt{x+8}$

$x-32 = 400 - 40\sqrt{x+8} + (x+8)$

$x-32 = 408 + x - 40\sqrt{x+8}$

$440 = -40\sqrt{x+8}$

$-11 = \sqrt{x+8}$

$121 = x+8$

$\boxed{113 = x}$

3. NCTM April 2018 #24

Given $6 < \sqrt{a} < 7$. Find an integer n such that $n < \sqrt[3]{a} < (n+1)$. Can you do this without a calculator?

$6 < a^{1/2} < 7$

$6^{2/3} < \left(a^{1/2}\right)^{2/3} < 7^{2/3}$

$6^{2/3} < a^{1/3} < 7^{2/3}$

$\sqrt[3]{36} < a^{1/3} < \sqrt[3]{49}$

$a^{1/2}$ to $a^{1/3}$

$\left(a^{1/2}\right)^{2/3} = a^{1/3}$

$\sqrt[3]{27} < \sqrt[3]{36} < \sqrt[3]{64}$

$3 < \sqrt[3]{36} < 4$

$\sqrt[3]{27} < \sqrt[3]{49} < \sqrt[3]{64}$

$3 < \sqrt[3]{49} < 4$

$3 < \sqrt[3]{36} < a^{1/3} < \sqrt[3]{49} < 4$

$3 < a^{1/3} < 4$

$\boxed{n = 3}$