October 2018 #19

Given three positive integers $a$, $b$, and $c$, such that $a^2 + b^2 = c^2$. Prove that $(2a + b + 2c)^2 + (a + 2b + 2c)^2 = (2a + 2b + 3c)^2$.

$$(2a + b + 2c)^2 + (a + 2b + 2c)^2 = (4a^2 + b^2 + 4c^2 + 4ab + 8ac + 4bc)$$
$$+ (a^2 + 4b^2 + 4c^2 + 4ab + 4ac + 8bc)$$

$$= (4a^2 + 4b^2 + (a^2 + b^2) + 8c^2 + 8ab + 12ac + 12bc)$$

$$= (4a^2 + 4b^2 + c^2 + 8c^2 + 8ab + 12ac + 12bc)$$

$$= (4a^2 + 4b^2 + 9c^2 + 8ab + 12ac + 12bc)$$

$$= (2a + 2b + 3c)^2$$

April 2014 #12

If

$f(x) = x^2 + bx + c,$
$f(1) = 9$, and
$f(3) - f(2) = 8,$

find $f(4)$.

$$f(1) = 9 = 1^2 + b(1) + c$$
$$9 = 1 + b + c$$

$$f(3) = 9 + 3b + c \qquad f(2) = 4 + 2b + c$$

$$9 + 3b + c - (4 + 2b + c) = 8$$
$$5 + b = 8$$
$$\boxed{b = 3}$$

$$9 = 1 + 3 + c$$
$$\boxed{5 = c}$$

$$f(x) = x^2 + 3x + 5$$

$$f(4) = 16 + 12 + 5$$
$$= \boxed{33}$$

September 2014 #14

Find the sum of the solutions to the equation $3(3^{2x}) - 28(3^{x}) = -9$. ~~(Use a calculator in the final step.)~~

$y = 3^x$

$3y^2 - 28y + 9 = 0$

$(3y - 1)(y - 9) = 0$

$y = 1/3 \qquad y = 9$

$3^x = 1/3 \qquad 3^x = 9$

$\boxed{x = -1 \qquad x = 2}$

October 2014 #13

**Solve the following system of equations:**

$$\begin{cases} \dfrac{3}{x+1} + \dfrac{5}{y-2} = 1 \\ \dfrac{6}{x+1} + \dfrac{1}{y-2} = 5 \end{cases}$$

$-2$

$\dfrac{-6}{x+1} - \dfrac{10}{y-2} = -2$

$+ \quad \dfrac{6}{x+1} + \dfrac{1}{y-2} = 5$

$-\dfrac{9}{y-2} = 3$

$-9 = 3y - 6$

$-3 = 3y$

$\boxed{-1 = y}$

$\dfrac{3}{x+1} + \dfrac{5}{-1-2} = 1$

$\dfrac{3}{x+1} = 1 + 5/3$

$3 = 8/3\,(x+1)$

$3 = 8/3x + 8/3$

$\dfrac{1}{3} = \dfrac{8}{3}x$

$\boxed{\dfrac{1}{8} = x}$

Math Team Oct 2017 #3

**If $p*q = (p-q)(p+q)$ and $p\Delta q = (p+q)^2 - 2pq$, find $(3 * 4)(3 \Delta 4)$.**

$3*4 = (3-4)(3+4)$

$= -7$

$3\Delta 4 = (3+4)^2 - 2(3)(4)$

$= 49 - 24$

$= 25$

$-7(25)$

$\boxed{-175}$