10.1 and 10.2 Practice

Name

1. If x and y are positive real numbers, which of the following conditions guarantees that the infinite series $\sum_{n=0}^{\infty}\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right)^n$ converges?

(A) $x > y$

(B) $x > y^2$

(C) $x < y$

(D) $x < y^2$

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2. Let x be a real number. Which of the following statements about the infinite series $\sum_{k=0}^{\infty} e^{kx}$ is true?

(A) The sum of the series is $\frac{1}{1-e^x}$ if $x < 0$.

(B) The sum of the series is $\frac{1}{1-e^x}$ if $x > 0$.

(C) The sum of the series is $\frac{e^x}{1-e^x}$ if $x < 0$.

(D) The sum of the series is $\frac{e^x}{1-e^x}$ if $x > 0$.

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3. Let f be the function defined by $f(x) = \frac{1}{1-x}$. Which of the following is the Maclaurin series for f' ?

10.1 and 10.2 Practice

(A) $\sum_{n=1}^{\infty} x^{n-1}$

(B) $\sum_{n=1}^{\infty} nx^{n-1}$

(C) $\sum_{n=1}^{\infty} (-1)^n x^{n-1}$

(D) $\sum_{n=1}^{\infty} (-1)^n nx^{n-1}$

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4. The second-degree Taylor polynomial for $f(x) = \frac{\cos x}{1-x}$ about $x = 0$ is

(A) $1 + \frac{x^2}{2}$

(B) $1 + x^2$

(C) $1 + x + \frac{x^2}{2}$

(D) $1 + x + x^2$

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5. Let f be the function with $f(0) = 0$ and derivative $f'(x) = \dfrac{1}{1+x^7}$. Which of the following is the Maclaurin series for f ?

(A) $x - \frac{x^8}{8} + \frac{x^{15}}{15} - \frac{x^{22}}{22} + \cdots$

(B) $x + \frac{x^8}{8} + \frac{x^{15}}{15} + \frac{x^{22}}{22} + \cdots$

(C) $-7x^6 + 14x^{13} - 21x^{20} + \cdots$

(D) $7x^6 + 14x^{13} + 21x^{20} + \cdots$

10.1 and 10.2 Practice

6. If $a_k = (-1)^k$ for $k = 0, 1, 2, \ldots$, which of the following statements about the infinite series $\sum_{k=0}^{\infty} a_k$ is true?

(A) The series converges and has sum 0.

(B) The series converges and has sum -1.

(C) The series converges and has sum 1.

(D) The series diverges.

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7. The infinite series $\sum_{k=1}^{\infty} a_k$ has *n*th partial sum $S_n = \frac{n}{3n+1} for, n \geq 1.$ What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) The series diverges

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8. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

10.1 and 10.2 Practice

(A) 1

(B) 2

(C) 4

(D) 6

(E) The series diverges.

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9. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

(A) $\frac{-2}{e^2-2e}$

(B) $\frac{-2}{e^2+2e}$

(C) $\frac{-2}{e+2}$

(D) $\frac{e}{e+2}$

(E) The series diverges.

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10. What is the value of $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$?

10.1 and 10.2 Practice

(A) $-\frac{15}{8}$

(B) $-\frac{9}{8}$

(C) $-\frac{3}{8}$

(D) $\frac{9}{8}$

(E) $\frac{15}{8}$

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11. Let *f* be a function with second derivative f''(x) = $\sqrt{1+3x}$. The coefficient of x^3 in the Taylor series for *f* about *x = 0* is

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

(E) $\frac{3}{2}$

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12. What are all values of *x* for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

10.1 and 10.2 Practice

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

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13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$

(B) 3

(C) $\sqrt{3}$

(D) $\frac{\sqrt{3}}{2}$

(E) 0

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14. The power series $\sum_{n=0}^{\infty} a_n(x-3)^n$ converges at $x = 5$. Which of the following must be true?

10.1 and 10.2 Practice

(A) The series diverges at $x = 0$.

(B) The series diverges at $x = 1$.

(C) The series converges at $x = 1$.

(D) The series converges at $x = 2$.

(E) The series converges at $x = 6$.

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15. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \leq 3$

(B) $-3 \leq x \leq 3$

(C) $-2 < x < 4$

(D) $-2 \leq x < 4$

(E) $0 \leq x \leq 2$

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16. Which of the following is the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n}$?

10.1 and 10.2 Practice

(A) -4 < x < 0

(B) $-4 \leq x < 0$

(C) -2 < x < 0

(D) $-2 \leq x < 0$

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17. What is the radius of convergence of the Maclaurin series for $\frac{2x}{1+x^2}$?

(A) 1/2

(B) 1

(C) 2

(D) infinite

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18. What is the radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{2 \cdot 3^{n+1}}$?

10.1 and 10.2 Practice

(A) $\frac{1}{3}$

(B) $\frac{3}{2}$

(C) 3

(D) 4

(E) 6

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19. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for *f* aboutx=0 ?

(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$

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20. A function *f* has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \cdots$. Which of the following is an expression for $f(x)$?

10.1 and 10.2 Practice

(A) $-3x \sin x + 3x^2$

(B) $-\cos\left(x^2\right) + 1$

(C) $-x^2 \cos x + x^2$

(D) $x^2 e^x - x^3 - x^2$

(E) $e^{x^2} - x^2 - 1$

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21. Which of the following is the Maclaurin series for $\frac{1}{(1-x)^2}$?

(A) $1 - x + x^2 - x^3 + \cdots$

(B) $1 - 2x + 3x^2 - 4x^3 + \cdots$

(C) $1 + 2x + 3x^2 + 4x^3 + \cdots$

(D) $1 + x^2 + x^4 + x^6 + \cdots$

(E) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$

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22. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

10.1 and 10.2 Practice

(A) $\frac{1}{1440}$

(B) $\frac{81}{160}$

(C) $\frac{9}{4}$

(D) $\frac{9}{2}$

(E) $\frac{27}{2}$

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23. A series expansion of $\frac{\sin t}{t}$ is

(A) $1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

(B) $\frac{1}{t}-\frac{t}{2!}+\frac{t^3}{4!}-\frac{t^5}{6!}+\cdots$

(C) $1+\frac{t^2}{3!}+\frac{t^4}{5!}+\frac{t^6}{7!}+\cdots$

(D) $\frac{1}{t}+\frac{t}{2!}+\frac{t^3}{4!}+\frac{t^5}{6!}+\cdots$

(E) $t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots$

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24. Which of the following is a power series expansion of $\frac{e^x+e^{-x}}{2}$?

10.1 and 10.2 Practice

(A) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots+\frac{x^{2n}}{(2n)!}+\cdots$

(B) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+\cdots$

(C) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots+\frac{x^{2n+1}}{(2n+1)!}+\cdots$

(D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\cdots$

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25. What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x=0$?

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

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26. Let *f* be the function given by $f(x)=\frac{1}{(2+x)}$. What is the coefficient of x^3 in the Taylor series for f about $x = 0$?

10.1 and 10.2 Practice

(A) -3/8

(B) -1/8

(C) -1/16

(D) 1/24

(E) 1/16

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27. The Maclaurin series for the function *f* is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3

(B) $-\frac{3}{7}$

(C) $\frac{4}{7}$

(D) $\frac{13}{16}$

(E) 4

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28. Which of the following are the first four nonzero terms of the Maclaurin series for the function g defined by $g(x) = (1+x)e^{-x}$?

10.1 and 10.2 Practice

(A) $1 + 2x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \cdots$

(B) $1 + 2x + \frac{3}{2}x^2 + \frac{5}{6}x^3 + \cdots$

(C) $1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \cdots$

(D) $1 - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{8}x^4 + \cdots$