10.1 and 10.2 Practice

Name

1. If x and y are positive real numbers, which of the following conditions guarantees that the infinite series $\sum_{n=0}^{\infty}\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right)^n$ converges?

geo

$n=0 \quad n=1 \quad n=2 \quad n=3$

$\frac{x}{y} + \frac{x^2}{y^3} + \frac{x^3}{y^5} + \frac{x^4}{y^7} + \dots$

(A) $x > y$

(B) $x > y^2$

(C) $x < y$

(D) $x < y^2$

geometric

$r = \frac{x}{y^2}$

$\left|\frac{x}{y^2}\right| < 1$

$* \; x/y^2 > 0$

$\frac{x}{y^2} < 1$

$x < y^2$

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2. Let x be a real number. Which of the following statements about the infinite series $\sum_{k=0}^{\infty} e^{kx}$ is true?

(A) The sum of the series is $\frac{1}{1-e^x}$ if $x < 0$.

(B) The sum of the series is $\frac{1}{1-e^x}$ if $x > 0$.

(C) The sum of the series is $\frac{e^x}{1-e^x}$ if $x < 0$.

(D) The sum of the series is $\frac{e^x}{1-e^x}$ if $x > 0$.

$\sum_{K=0}^{\infty} (e^x)^K$

geo $\quad r = e^x$

$a_1 = 1$

$\frac{1}{1-e^x}$

$|e^x| < 1$

$e^x < 1$

e^x always pos

need e^x to be a fraction so x needs to be neg.

$x < 0$

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3. Let f be the function defined by $f(x) = \frac{1}{1-x}$. Which of the following is the Maclaurin series for f'?

$(1-x)^{-1}$

$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots + \sum_{n=0}^{\infty} x^n$

$f'(x) = \sum_{n=0}^{\infty} n x^{n-1}$

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(A) $\sum_{n=1}^{\infty} x^{n-1}$

(B) $\sum_{n=1}^{\infty} nx^{n-1}$

(C) $\sum_{n=1}^{\infty} (-1)^n x^{n-1}$

(D) $\sum_{n=1}^{\infty} (-1)^n nx^{n-1}$

4. The second-degree Taylor polynomial for $f(x) = \frac{\cos x}{1-x}$ about $x = 0$ is

(A) $1 + \frac{x^2}{2}$

(B) $1 + x^2$

(C) $1 + x + \frac{x^2}{2}$

(D) $1 + x + x^2$

$\frac{1}{1-x} = 1 + x + x^2 + \ldots$ $\cos x = 1 - \frac{x^2}{2} + \ldots$

$\cos x\left(\frac{1}{(1-x)}\right) \approx \left(1 - \frac{x^2}{2}\right)\left(1 + x + x^2\right)$

$= 1 + x + \underline{x^2} - \underline{\frac{1}{2}x^2} - \frac{1}{2}x^3 - \frac{1}{2}x^4$

$P_2(x) = 1 + x + \frac{1}{2}x^2$

* only need second order so use as many terms as needed to get complete x^2 term

5. Let f be the function with $f(0) = 0$ and derivative $f'(x) = \frac{1}{1+x^7}$. Which of the following is the Maclaurin series for f?

$r = -x^7 \quad a_1 = 1$

(A) $x - \frac{x^8}{8} + \frac{x^{15}}{15} - \frac{x^{22}}{22} + \cdots$

(B) $x + \frac{x^8}{8} + \frac{x^{15}}{15} + \frac{x^{22}}{22} + \cdots$

(C) $-7x^6 + 14x^{13} - 21x^{20} + \cdots$

(D) $7x^6 + 14x^{13} + 21x^{20} + \cdots$

$f'(x) = 1 - x^7 + x^{14} - x^{21} + x^{28} + \ldots$

$\int f'(x)\,dx = \int (1 - x^7 + x^{14} - x^{21} + x^{28} + \ldots)\,dx$

$f(x) = x - \frac{1}{8}x^8 + \frac{1}{15}x^{15} - \frac{1}{22}x^{22} + \ldots$

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6. If $a_k = (-1)^k$ for $k = 0, 1, 2, \dots$, which of the following statements about the infinite series $\sum_{k=0}^{\infty} a_k$ is true?

(A) The series converges and has sum 0.

(B) The series converges and has sum -1.

(C) The series converges and has sum 1.

(D) The series diverges.

Partial sums

$s_1 = -1$

$s_2 = -1 + 1 = 0$

$s_3 = -1 + 1 - 1 = -1$

⋮

$-1, 0, -1, 0, -1 \dots$

$\lim_{k \to \infty}$ of Partial sums $=$ DNE diverges

7. Consider the sequence $a_k = \sqrt{k+1} - \sqrt{k} = \frac{1}{\sqrt{k+1}+\sqrt{k}}$ and the infinite series $\sum_{k=1}^{\infty} a_k$. Which of the following is true?

(A) $\lim_{k\to\infty} a_k = 0$ and $\sum_{k=1}^{\infty} a_k$ converges.

(B) $\lim_{k\to\infty} a_k \neq 0$ and $\sum_{k=1}^{\infty} a_k$ converges.

(C) $\lim_{k\to\infty} a_k = 0$ and $\sum_{k=1}^{\infty} a_k$ diverges.

(D) $\lim_{k\to\infty} a_k \neq 0$ and $\sum_{k=1}^{\infty} a_k$ diverges.

partial sums

$s_1 = \sqrt{2} - 1$

$s_2 = \sqrt{3} - \sqrt{2}$

$s_3 = 2 - \sqrt{3}$

$s_4 = \sqrt{5} - 2$

$s_5 = \sqrt{6} - \sqrt{5}$

$s_6 = \sqrt{7} - \sqrt{6}$

getting smaller & smaller approaching 0

* check on calc

calc active

8. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = \frac{n}{3n+1}$ for, $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$?

$s_1 = \frac{1}{4} = 0.25$

$s_2 = \frac{2}{7} \approx 0.2857$

$s_3 = \frac{3}{10} = 0.3$

$s_4 = \frac{4}{13} \approx 0.3077$

$s_5 = \frac{5}{16} \approx 0.3125$

$s_6 = \frac{6}{19} \approx 0.3157$

$s_7 = \frac{7}{22} = 0.3\overline{18}$

$s_8 = 8/25 = 0.32$

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(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) The series diverges

0.25, 0.2857, 0.3, 0.3077, 0.3125, 0.3157,

$0.3\overline{18}$, 0.32

$\lim_{n\to\infty}$ of partial sums $= \frac{1}{3}$

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9. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

geo

(A) 1

(B) 2

(C) 4

(D) 6

(E) The series diverges.

$\sum_{n=1}^{\infty} 2 \cdot \frac{2^n}{3^n} = \sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^n$

$a_1 = \frac{4}{3}$ $\quad r = \frac{2}{3} < 1$ ✓

$\frac{4/3}{1 - 2/3} = \frac{4/3}{1/3}$

$= 4$

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10. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?

geo

$\sum_{n=1}^{\infty} \frac{(-2)^n}{e \cdot e^n} = \sum_{n=1}^{\infty} \frac{1}{e}\left(\frac{-2}{e}\right)^n$

$a_1 = -\frac{2}{e^2}$ $\quad r = -2/e$

$|-2/e| < 1$ ✓

$\frac{-2/e^2}{1 + 2/e} = \frac{-2/e^2}{\frac{e+2}{e}}$

$= -\frac{2}{e^2} \cdot \frac{e}{e+2} = -\frac{2}{e^2+2e}$

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(A) $\frac{-2}{e^2-2e}$

(B) $\frac{-2}{e^2+2e}$

(C) $\frac{-2}{e+2}$

(D) $\frac{e}{e+2}$

(E) The series diverges.

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11. What is the value of $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$?

$r = \frac{-3}{5}$ $|-3/5| < 1$ ✓

$a_1 = 9/5$

$\frac{9/5}{1+3/5} = \frac{9/5}{8/5} = \frac{9}{8}$

(A) $-\frac{15}{8}$

(B) $-\frac{9}{8}$

(C) $-\frac{3}{8}$

(D) $\frac{9}{8}$

(E) $\frac{15}{8}$

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12. Let f be a function with second derivative $f''(x) = \sqrt{1+3x}$. The coefficient of x^3 in the Taylor series for f about $x = 0$ is

$f'''(x) = \frac{1}{2}(1+3x)^{-1/2}(3)$

$f'''(0) = \frac{3}{2}\frac{1}{\sqrt{1}}$

$= \frac{3}{2}$

$\frac{f'''(0)}{3!}(x-0)^3$

$= \frac{3/2}{6}x^2 = \frac{1}{4}x^2$

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(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

(E) $\frac{3}{2}$

13. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges? geo

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

$r = \frac{2}{x^2+1}$

$-1 < \frac{2}{x^2+1} < 1$

$-1 < \frac{2}{x^2+1}$ and $\frac{2}{x^2+1} < 1$

$-x^2 - 1 < 2$

$-x^2 < 3$

$x^2 > -3$

no solution

$2 < x^2 + 1$

$x^2 > 1$

$|x| > 1$ $\quad x > 1$ or $x < -1$

14. What is the radius of convergence of the series $\sum_{n=0}^{\infty}\frac{(x-4)^{2n}}{3^n}$? geo

$= \sum_{n=0}^{\infty}\frac{((x-4)^2)^n}{3^n}$ $\qquad r = \frac{(x-4)^2}{3}$

$-1 < \frac{(x-4)^2}{3} < 1$

$-3 < (x-4)^2 < 3$

$-3 < (x-4)^2$ and $(x-4)^2 < 3$

no solution $\qquad |x-4| < \sqrt{3}$

$x - 4 < \sqrt{3}$ and $x - 4 > -\sqrt{3}$

$x < \sqrt{3} + 4$ $\qquad x > -\sqrt{3} + 4$

$-\sqrt{3} + 4 < x < \sqrt{3} + 4$

* radius * centered @ $x = 4$

10.1 and 10.2 Practice

$-\sqrt{3}$ $\sqrt{3}$

$4-\sqrt{3}$ 4 $4+\sqrt{3}$

radius = $\sqrt{3}$

(A) $2\sqrt{3}$

(B) 3

(C) $\sqrt{3}$

(D) $\frac{\sqrt{3}}{2}$

(E) 0

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15. The power series $\sum_{n=0}^{\infty} a_n(x-3)^n$ converges at $x=5$. Which of the following must be true?

centered @ $x = 3$

(A) The series diverges at $x=0$.
could be T

(B) ~~The series diverges at $x=1$.~~
F

(C) The series converges at $x=1$.
not necessarily true
↳ convergence at endpoints not guaranteed

(D) The series converges at $x=2$. T

(E) The series converges at $x=6$.
could be true

2 converges

3 5

so radius of convergence > 2

think of geo interval of convergence $-1<r<1$

we will learn how to check endpoints for convergence in 10.5

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16. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is geo

$r = \frac{x-1}{3}$

$-1 < \frac{x-1}{3} < 1$

$-3 < x-1 < 3$

$-2 < x < 4$

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(A) $-3 < x \leq 3$

(B) $-3 \leq x \leq 3$

(C) $-2 < x < 4$

(D) $-2 \leq x < 4$

(E) $0 \leq x \leq 2$

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17. Which of the following is the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n}$? geo

(A) -4 < x < 0

(B) $-4 \leq x < 0$

(C) -2 < x < 0

(D) $-2 \leq x < 0$

$r = \frac{x+2}{2}$

$-1 < \frac{x+2}{2} < 1$

$-2 < x + 2 < 2$

$-4 < x < 0$

---

18. What is the radius of convergence of the Maclaurin series for $\frac{2x}{1+x^2}$? geo

$a_1 = 2x \quad r = -x^2$

$-1 < -x^2 < 1$

$1 > x^2 > -1$

$x^2 < 1$ and $x^2 > -1$

no sol

$|x| < 1$

$x < 1$ and $x > -1$

$-1 < x < 1$ radius = 1

10.1 and 10.2 Practice

(A) 1/2

(B) 1

(C) 2

(D) infinite

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19. What is the radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{2 \cdot 3^{n+1}}$? $= \sum_{n=0}^{\infty} \frac{1}{2 \cdot 3} \frac{(x-4)^n}{3^n}$

(A) $\frac{1}{3}$

(B) $\frac{3}{2}$

(C) 3

(D) 4

(E) 6

$r = \frac{x-4}{3}$

$-1 < \frac{x-4}{3} < 1$

$-3 < x-4 < 3$

$1 < x < 7$

radius = 3

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20. If $f(x) = x \sin(2x)$, which of the following is the Taylor series for f aboutx=0 ?

$\sin x = 1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\sin 2x = 1 - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{(2x)^7}{7!} + \ldots + \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}$

$x \sin 2x = x - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{(2^7)x^8}{7!} + \ldots + \frac{(-1)^n 2^{2n+1} x^{2n+2}}{(2n+1)!}$

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(A) $x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \cdots$

(B) $x - \frac{4x^3}{2!} + \frac{16x^5}{4!} - \frac{64x^7}{6!} + \cdots$

(C) $2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!} + \cdots$

(D) $2x^2 - \frac{2x^4}{3!} + \frac{2x^6}{5!} - \frac{2x^8}{7!} + \cdots$

(E) $2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!} + \cdots$

$\leftarrow$ no 0 terms would expect to see w/ sine & cosine

21. A function f has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x \sin x + 3x^2$

(B) $-\cos(x^2) + 1$

(C) $-x^2 \cos x + x^2$

(D) $x^2 e^x - x^3 - x^2$

(E) $e^{x^2} - x^2 - 1$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^x x^2 = \underbrace{x^2 + x^3}_{\text{don't want}} + \frac{x^4}{2!} + \frac{x^5}{3!} + \ldots$$

$$e^x x^2 - x^2 - x^3 = \frac{x^4}{2!} + \frac{x^5}{3!} + \ldots$$

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22. Which of the following is the Maclaurin series for $\frac{1}{(1-x)^2}$?

$$\frac{1}{(1-x)^2} = \frac{1}{(1-x)} \frac{1}{(1-x)}$$

$$\frac{1}{(1-x)} = 1 + x + x^2 + x^3 + \ldots$$

$$(1 + x + x^2)(1 + x + x^2)$$

$$1 + x + x^2 + x + x^2 + x^3 + x^2 + x^3 + x^4$$

$1 + 2x + 3x^2 + \ldots$ dont have all x^3 and x^4 terms b/c only used

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(A) $1 - x + x^2 - x^3 + \cdots$

(B) $1 - 2x + 3x^2 - 4x^3 + \cdots$

(C) $1 + 2x + 3x^2 + 4x^3 + \cdots$

(D) $1 + x^2 + x^4 + x^6 + \cdots$

(E) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$

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23. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

(A) $\frac{1}{1440}$

(B) $\frac{81}{160}$

(C) $\frac{9}{4}$

(D) $\frac{9}{2}$

(E) $\frac{27}{2}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$e^{3x^2} = 1 + 3x^2 + \frac{9x^4}{2!} + \frac{27x^6}{3!} + \ldots$$

$$\frac{e^{3x^2}}{2} = \frac{1}{2} + \frac{3}{2} + \frac{9x^4}{4} + \frac{27}{3!\,2}x^6 + \ldots$$

$$\frac{27}{3!\,2} = \frac{27}{6 \cdot 2} = \frac{27}{12} = \frac{9}{4}$$

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24. A series expansion of $\frac{\sin t}{t}$ is

$$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \ldots$$

$$\frac{\sin t}{t} = 1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \ldots$$

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(A) $1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

(B) $\frac{1}{t}-\frac{t}{2!}+\frac{t^3}{4!}-\frac{t^5}{6!}+\cdots$

(C) $1+\frac{t^2}{3!}+\frac{t^4}{5!}+\frac{t^6}{7!}+\cdots$

(D) $\frac{1}{t}+\frac{t}{2!}+\frac{t^3}{4!}+\frac{t^5}{6!}+\cdots$

(E) $t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots$

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25. Which of the following is a power series expansion of $\frac{e^x+e^{-x}}{2}$?

(A) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots+\frac{x^{2n}}{(2n)!}+\cdots$

(B) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+\cdots$

(C) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots+\frac{x^{2n+1}}{(2n+1)!}+\cdots$

(D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\cdots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!}$

$e^x + e^{-x} = 2 + \frac{2x^2}{2!} + \frac{2x^4}{4!}$

$\frac{e^x+e^{-x}}{2} = 1 + \frac{x^2}{2} + \frac{x^4}{4!}$

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26. What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x=0$?

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\sin^2 x = (\sin x)(\sin x)$

$= x^2 + \ldots$

$\uparrow$

$x \cdot x$

and no other x^2 term is constructed when foiling

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(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

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27. Let *f* be the function given by $f(x) = \frac{1}{(2+x)}$. What is the coefficient of x^3 in the Taylor series for f about $x = 0$?

$\frac{1}{2(1+\frac{1}{2}x)}$

(A) -3/8

(B) -1/8

$\frac{\frac{1}{2}}{1+\frac{1}{2}x} = \frac{1}{2} - \frac{1}{4}x + \frac{1}{8}x^2 - \frac{1}{16}x^3$

$r = -\frac{1}{2}x$

(C) -1/16

(D) 1/24

This way doesn't work! B/c centered @ $x = 1$ not $x = 0$

$\frac{1}{1-(-x-1)} = 1 + (-x-1) + (-x-1)^2 + \ldots$

(E) 1/16

$= -x + x^2 - 2x + 1$

$r = -x-1$

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28. The Maclaurin series for the function *f* is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

$a_1 = 1 \quad r = -x/4$ geo!

$f(x) = \frac{1}{1-(-x/4)} = \frac{1}{1+x/4} = \frac{1}{\frac{4+x}{4}} = \frac{4}{4+x}$

$f(3) = \frac{4}{7}$

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(A) -3

(B) $-\frac{3}{7}$

(C) $\frac{4}{7}$

(D) $\frac{13}{16}$

(E) 4

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29. Which of the following are the first four nonzero terms of the Maclaurin series for the function g defined by $g(x) = (1+x)e^{-x}$? $= e^{-x} + xe^{-x}$

(A) $1 + 2x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \cdots$

(B) $1 + 2x + \frac{3}{2}x^2 + \frac{5}{6}x^3 + \cdots$

(C) $1 - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \cdots$

(D) $1 - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{8}x^4 + \cdots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!}$

$xe^{-x} = x - x^2 + \frac{x^3}{2} - \frac{x^4}{3!} + \ldots$

$e^{-x} + xe^{-x} = 1 - \frac{x^2}{2} + \frac{x^3}{2} - \frac{x^3}{6} + \ldots$

$= 1 - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \ldots$