Error Bound

Name

1. If the infinite series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n}$, is approxiately by $P_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{2}{n}$, what is the least value of *k* for which the alternating series error bound guarantees that $|S - P_k| < \frac{3}{100}$?

(A) 64

(B) 66

(C) 68

(D) 70

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2. The Taylor series for a function *f* about *x* = 0 is given by $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} x^{2n}$ and converges to *f* for all real numbers *x*. If the fourth-degree Taylor polynomial for *f* about *x* = 0 is used to approximate $f\left(\frac{1}{2}\right)$ alternating series error bound?

(A) $\frac{1}{2^4 \cdot 5!}$

(B) $\frac{1}{2^5 \cdot 6!}$

(C) $\frac{1}{2^6 \cdot 7!}$

(D) $\frac{1}{2^{10} \cdot 11!}$

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3. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges to *S*. Based on the alternating series error bound, what is the least number of terms in the series that must be summed to guarantee a partial sum that is within 0.03 of *S* ?

Error Bound

(A) 34

(B) 333

(C) 1111

(D) 9999

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4. The function *f* has derivatives of all orders for all real numbers, and $f^{(4)}(x)=e^{\sin x}$. If the third-degree Taylor polynomial for *f* about x=0 is used to approximate *f* on the interval [0,1] , what is the Lagrange error bound for the maximum error bound for the maximum error on the interval [0,1] ?

(A) 0.019

(B) 0.097

(C) 0.113

(D) 0.399

(E) 0.417

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5. Let *f* be a function that has derivatives of all orders for all real numbers, and let $P_3(x)$ be the third-degree Taylor polynomial for *f* about $x = 0$. The Taylor series for *f* about $x = 0$ converges at $x = 1$, and $\left|f^{(n)}(x)\right| \leq \frac{n}{n+1} for, 1 \leq n \leq 4$ and all values of *x*. Of the following, which is the smallest value of *k* for which the Lagrange error bound guarantees that $|f(1) - P_3(1)| \leq k$?

Error Bound

(A) $\frac{4}{5}$

(B) $\frac{4}{5} \cdot \frac{1}{4!}$

(C) $\frac{4}{5} \cdot \frac{1}{3!}$

(D) $\frac{3}{4} \cdot \frac{1}{4!}$

(E) $\frac{3}{4} \cdot \frac{1}{3!}$

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6. If the series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ is approximated by the partial sum $S_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{1}{n^2}$, what is the least value
of k for which the alternating series error bound guarantees that $\left| S - S_k \right| \leq \frac{9}{10{,}000}$?

(A) 31

(B) 32

(C) 33

(D) 34

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7. If the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$ is approximated by the partial sum with 15 terms, what is the alternating series error bound?

Error Bound

(A) $\frac{1}{15}$

(B) $\frac{1}{16}$

(C) $\frac{1}{31}$

(D) $\frac{1}{33}$

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8. The series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k$ converges to S and $0 < a_{k+1} < a_k$ for all k. If $S_n = \sum_{k=1}^{n}(-1)^{k+1}a_k$ is the $n\text{th}$ partial sum of the series, which of the following statements must be true?

(A) $\lim_{n\to\infty} S_n = 0$

(B) $\lim_{n\to\infty} a_n = S$

(C) $|S - S_{10}| \le a_{16}$

(D) $|S - S_{15}| \le a_{16}$

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9. The Taylor series for a function f about $x = 2$ is given by $\sum_{n=0}^{\infty}(-1)^n \frac{3n+1}{2^n}(x-2)^n$ and converges to f for $0 < x < 4$. If the third-degree Taylor polynomial for f about $x = 2$ is used to approximate $f\left(\frac{9}{4}\right)$, what is the alternating series error bound?

(A) $\frac{10}{8 \cdot 4^3}$

(B) $\frac{1}{24 \cdot 4^4}$

(C) $\frac{13}{16 \cdot 4^4}$

(D) $\frac{13 \cdot 9^4}{16 \cdot 4^4}$

10.

$\max\limits_{0 \le x \le 1.2} \left\|f^{(5)}(x)\right\| = 8.4$	$\max\limits_{0 \le x \le 1.2} \left\|f^{(6)}(x)\right\| = 58.8$	$\max\limits_{0 \le x \le 1.2} \left\|f^{(7)}(x)\right\| = 411.8$

Let $P(x)$ be the fifth-degree Taylor polynomial for a function f about $x = 0$. Information about the maximum of the absolute value of selected derivatives of f over the interval $0 \le x \le 1.2$ is given in the table above. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(1.2) - P(1.2)| \le k$?

(A) 0.082

(B) 0.174

(C) 0.244

(D) 0.293

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11. Let f be a polynomial function with nonzero coefficients such that $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$. $T_3(x)$ is the third-degree Taylor polynomial for f about $x = c$ such that $T_3(x) = b_0 + b_1(x - c) + b_2(x - c)^2 + b_3(x - c)^3$. Based on use of the Lagrange error bound, $f(x) - T_3(x)$ must equal which of the following?

Error Bound

(A) 0

(B) $(x-c)^4$

(C) $a_4(x-c)^4$

(D) $4! \cdot a_4(x-c)^4$