Error Bound

Name

calc

2\. If the infinite series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n}$, is approximately by $P_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{2}{n}$, what is the least value of k for which the alternating series error bound guarantees that $|S - P_k| < \frac{3}{100}$?

a_{n+1}

(A) 64

(B) 66

(C) 68

(D) 70

a) $a_{65} = \left| (-1)^{66}\, 2/65 \right| = \frac{2}{65} > \frac{3}{100}$

b) $a_{67} = \left| (-1)^{68}\, 2/67 \right| = \frac{2}{67} < \frac{3}{100}$ ✓

$f(x) = \frac{x^2}{3!} - \frac{x^4}{5!} + \frac{x^6}{7!}$

$P_4(x) = \frac{x^2}{3!} - \frac{x^4}{5!}$

3. The Taylor series for a function f about $x = 0$ is given by $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!} x^{2n}$ and converges to f for all real numbers x. If the fourth-degree Taylor polynomial for f about $x = 0$ is used to approximate $f\left(\frac{1}{2}\right)$ alternating series error bound?

(A) $\frac{1}{2^4 \cdot 5!}$

(B) $\frac{1}{2^5 \cdot 6!}$

(C) $\frac{1}{2^6 \cdot 7!}$

(D) $\frac{1}{2^{10} \cdot 11!}$

$\text{error} = \left| f(1/2) - P_4(1/2) \right| \leq \left| \frac{(1/2)^6}{7!} \right|$

$= \left| \frac{1}{2^6 \, 7!} \right|$

* not 5^{th} term following term after 4^{th} degree

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4. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges to S. Based on the alternating series error bound, what is the least number of terms in the series that must be summed to guarantee a partial sum that is within 0.03 of S?

a_{n+1}

(A) 34

(B) 333

(C) 1111

(D) 9999

$\text{error} = \left| f(x) - P_n(x) \right| < 0.03$ find n

a) $|a_{35}| = \frac{1}{\sqrt{35}} \approx 0.169$

b) $|a_{334}| = \frac{1}{\sqrt{334}} \approx 0.0547$

c) $|a_{1112}| = \frac{1}{\sqrt{1112}} \approx 0.0299$ ✓

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5. The function f has derivatives of all orders for all real numbers, and $f^{(4)}(x)=e^{\sin x}$. If the third-degree Taylor polynomial for f about x=0 is used to approximate f on the interval [0,1], what is the Lagrange error bound for the maximum error bound for the maximum error on the interval [0,1]?

$\text{error} = \left| f(x) - P_3(x) \right| \leq \left| \frac{f^{(4)}(c)\,(x-0)^4}{4!} \right|$

maximize $\to x=1$

$f^{(4)}(x) \leq e^{\sin(1)}$

$\leq \frac{e^{\sin(1)}(1)^4}{4!}$

$= 0.097$

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(A) 0.019

(B) 0.097

(C) 0.113

(D) 0.399

(E) 0.417

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6. Let f be a function that has derivatives of all orders for all real numbers, and let $P_3(x)$ be the third-degree Taylor polynomial for f about $x = 0$. The Taylor series for f about $x = 0$ converges at $x = 1$, and $\left|f^{(n)}(x)\right| \leq \frac{n}{n+1} for, 1 \leq n \leq 4$ and all values of x. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(1) - P_3(1)| \leq k$?

(A) $\frac{4}{5}$

(B) $\frac{4}{5} \cdot \frac{1}{4!}$

(C) $\frac{4}{5} \cdot \frac{1}{3!}$

(D) $\frac{3}{4} \cdot \frac{1}{4!}$

(E) $\frac{3}{4} \cdot \frac{1}{3!}$

error $= |f(1) - P_3(1)| \leq \left| \frac{f^{(4)}(c)}{4!} (1-0)^4 \right|$

maximize $f^{(4)}(c)$

$f^{(4)}(x) \leq \frac{4}{4+1}$

$= \frac{4}{5}$

$= \frac{4/5 \, (1)^4}{4!}$

$= \frac{4}{5} \cdot \frac{1}{4!}$

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7. If the series $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ is approximated by the partial sum $S_k = \sum_{n=1}^{k} (-1)^{n+1} \frac{1}{n^2}$, what is the least value of k for which the alternating series error bound guarantees that $\left|S - S_k\right| \leq \frac{9}{10{,}000}$?

a_{n+1}

0.0009

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(A) 31

(B) 32

(C) 33

(D) 34

a) $|a_{32}| = \frac{1}{32^2} \approx 0.000976$

b) $|a_{33}| = \frac{1}{33^2} \approx 0.000918$

c) $|a_{34}| = \frac{1}{34^2} \approx 0.000865$

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8. If the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$ is approximated by the partial sum with 15 terms, what is the alternating series error bound?

(A) $\frac{1}{15}$

(B) $\frac{1}{16}$

(C) $\frac{1}{31}$

(D) $\frac{1}{33}$

a_{n+1}

$\text{error} \le |a_{15+1}| = \frac{1}{2(16)+1}$

$= \frac{1}{33}$

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9. The series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k$ converges to S and $0 < a_{k+1} < a_k$ for all k. If $S_n = \sum_{k=1}^{n}(-1)^{k+1}a_k$ is the nth partial sum of the series, which of the following statements must be true?

terms are alternating and decreasing in absolute value

so alt. series error bound

$|\text{series} - s_n| \le |a_{n+1}|$

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(A) $\lim_{n\to\infty} S_n = 0$

(B) $\lim_{n\to\infty} a_n = S$

(C) $|S - S_{10}| \leq a_{16}$

(D) $|S - S_{15}| \leq a_{16}$

$f(x) = -\frac{4}{2}(x-2) + \frac{7}{4}(x-2)^2 - \frac{10}{8}(x-2)^3 + \frac{13}{16}(x-2)^4$

10. The Taylor series for a function f about $x = 2$ is given by $\sum_{n=0}^{\infty} (-1)^n \frac{3n+1}{2^n}(x-2)^n$ and converges to f for $0 < x < 4$. If the third-degree Taylor polynomial for f about $x = 2$ is used to approximate $f\left(\frac{9}{4}\right)$, what is the alternating series error bound?

(A) $\frac{10}{8 \cdot 4^3}$

(B) $\frac{1}{24 \cdot 4^4}$

(C) $\frac{13}{16 \cdot 4^4}$

(D) $\frac{13 \cdot 9^4}{16 \cdot 4^4}$

$$\text{error} = \left| f(9/4) - P_3(9/4) \right| \leq \left| \frac{13}{16}(x-2)^4 \right|$$

$$\leq \left| \frac{13}{16}(9/4 - 2)^4 \right|$$

$$= \left| \frac{13}{16}\left(\frac{1}{4}\right)^4 \right| = \frac{13}{16 \cdot 4^4}$$

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11.

$\max_{0\leq x\leq 1.2} \left|f^{(5)}(x)\right| = 8.4$	$\max_{0\leq x\leq 1.2} \left|f^{(6)}(x)\right| = 58.8$	$\max_{0\leq x\leq 1.2} \left|f^{(7)}(x)\right| = 411.8$

Let $P(x)$ be the fifth-degree Taylor polynomial for a function f about $x = 0$. Information about the maximum of the absolute value of selected derivatives of f over the interval $0 \leq x \leq 1.2$ is given in the table above. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(1.2) - P(1.2)| \leq k$?

$$\text{error} = \left| f(1.2) - P_5(1.2) \right| \leq \left| \frac{f^{(6)}(c)}{6!}(1.2-0)^6 \right|$$

$$= \frac{58.8}{6!}(1.2)^6$$

$= 0.244$

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(A) 0.082

(B) 0.174

(C) 0.244

(D) 0.293

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12. Let f be a polynomial function with nonzero coefficients such that $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. $T_3(x)$ is the third-degree Taylor polynomial for f about $x = c$ such that $T_3(x) = b_0 + b_1(x-c) + b_2(x-c)^2 + b_3(x-c)^3$. Based on use of the Lagrange error bound, $f(x) - T_3(x)$ must equal which of the following?

(A) 0

error = $|f(x) - T_3(x)| \le |a_4(x-c)^4|$

(B) $(x-c)^4$

(C) $a_4(x-c)^4$

(D) $4! \cdot a_4(x-c)^4$
