Convergence for a :

Geometric Series:

Maclaurin series for $\sin x$, $\cos x$, e^x :

Nth-Term Test for Divergence

Recall from 10.1 infinite series:

The nth-term Test for Divergence

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. *The converse of this **<u>is not</u>** true. If $\lim_{n\to\infty} a_n = 0$ the series **<u>could</u>** converge.

1. Determine if the series diverges:

   a. $\sum_{n=0}^{\infty} 2^n$

   b. $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

   c. $\sum_{n=1}^{\infty} \frac{1}{n}$

Direct Comparison Test

The direct comparison test is a tool that we can use to determine convergence for complicated, **positive** series by comparing them with simpler series.

Direct Comparison Test

Let $0 < a_n < b_n$ for all n

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges

2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

2. Determine convergence or divergence.

a. $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

b. $\sum_{n=0}^{\infty} \frac{x^{2n}}{(n!)^2}$

c. $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!+2}$

Definitions of Absolute and Conditional Convergence

A series **converges absolutely** (is absolutely convergent) if $\sum |a_n|$ converges.

3. Does the series converge absolutely or diverge?

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{\frac{n^2+n}{2}}}{3^n}$

b. $\displaystyle\sum_{n=0}^{\infty} \frac{(\sin x)^n}{n!}$

Ratio Test

Ratio test

Let $\sum a_n$ be a series with nonzero terms.

1. $\sum a_n$ converges absolutely if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$
2. $\sum a_n$ diverges if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$
3. The ratio test is inconclusive if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$

The ratio test is particularly useful for series that converge rapidly (i.e. factorials or exponentials).

4. Determine Convergence or Divergence:
   a. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$
   b. $\sum_{n=0}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

c. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

d. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$

$\sum_{n=0}^{\infty} c_n(x-a)^n$ always converges at $x = a$, which assures us of one point where the series must converge.

The Convergence Theorem for Power Series

There are three possibilities for $\sum_{n=0}^{\infty} c_n(x-a)^n$ with respect to convergence:

1. There is a positive number R such that the series diverges for $|x-a| > R$ but converges for $|x-a| < R$.
   *The series may or may not converge at either of the endpoints $x = a - R$ and $x = a + R$
2. The series converges for every x $(R = \infty)$
3. The series converges at $x = a$ and diverges elsewhere $(R = 0)$

*We find the radius of convergence by using the Ratio Test

Radius of Convergence for a power series tells you how far away from the center you can be and still find a good approximation

5. Find the Radius and Interval of Convergence for:
   a. $\sum_{n=0}^{\infty} n!x^n$
   b. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

c. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n\,2^n}$

d. $\sum_{n=0}^{\infty} \frac{(x-3)^{n+1}}{(n+1)\,4^{n+1}}$