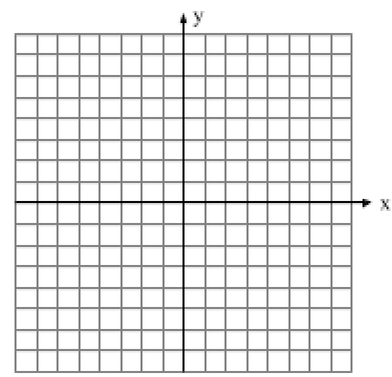
11.1 Parametric Functions
BC Calculus

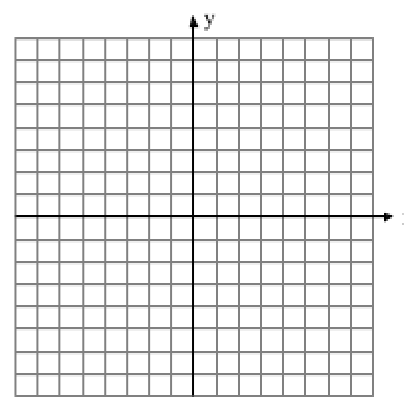
Parametric Function:

Instead of defining the points (x, y) on a planar curve by relating y directly to x, we can define both coordinates as functions of a parameter t.

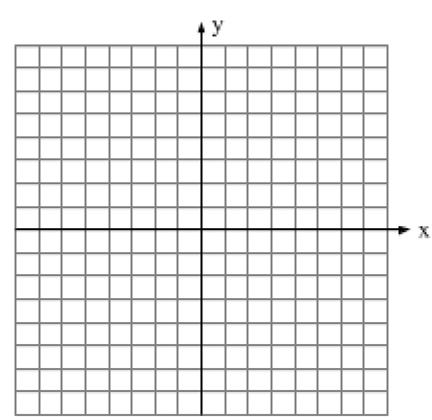
1. Make a table of values and sketch the curve, indicating the direction of your graph. Then, eliminate the parameter.
    a. $x = \sqrt{t-1},\ y = t + 2$



    b. $x = t^2 - 3$ and $y = 2t,\ -2 \le t \le 3$



    c. $x = 3 + 2\cos t,\ y = -1 + 3\sin t$



Slope and Concavity

For parametric function the slope of the curve is stil $\frac{dy}{dx}$, and the concavity still depends on $\frac{d^2y}{dx^2}$, so all that is needed is a way of differentiating with respect to x when everything is given in terms of t.

Parametric Differentiation Formulas

If x and y are both differentiable functions of t and if $\frac{dy}{dx} \neq 0$, then

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt},$$

If $y' = \frac{dy}{dx}$ is also differentiable functions of t, then

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right]\left(\frac{dt}{dx}\right) = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$$

2. Given the parametric functions $x = 2\sqrt{t}$ and $y = 3t^2 - 2t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

3. Given the parametric equations $x = 4\cos t$ and $y = 3\sin t$, write an equation of the tangent line to the curve at the point where $t = \frac{3\pi}{4}$

4. Find the equation of the line tangent to the curve given by $x = 2 - 3\cos\theta$ and $y = 3 + 2\sin\theta$ at the point $(-1, 3)$.

5. Find all points of of horizontal and vertical tangency given the parametric equations $x = t^2 + t$, $y = t^2 - 3t + 5$

6. Consider the curve defined parametrically by $x = t^2 - 5$ and $y = 2\sin t$ for $0 \le t \le \pi$.
    a. Sketch a graph of the curve in the viewing window $[-7, 7]$ by $[-4, 4]$. Indicate the direction in which it is traced.
    b. Find the highest point on the curve.
    c. Find all points of inflection on the curve.

Arc Length of a Parametric Curve
Let L by the length of a parametric curve that is traverse exactly once as t increases from t_1 to t_2.
If $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are continuous function of t, then

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

6. A particle moves along the smooth curve given by $x = t^2 + 1$ and $y = 4t^3 - 1$. How far did the particle travel between $t = 0$ and $t = 5$?