vector: a quantity having direction as well as magnitude

Velocity, Speed, Acceleration, and Direction of Motion
Suppose a particle moves along a smooth curve in the plane so that its position at any time t is $(x(t), y(t))$, where x and y are differentiable functions of t.

1. The particle's **position vector** is $r(t) = \langle x(t), y(t) \rangle$
2. The particle's **velocity vector** is $v(t) = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$
3. The particle's **speed** is the magnitude of **v**, denoted $|v|$. Speed is a scalar not a vector.
   a magnitude (how much of something)
4. The particle's **acceleration vector** is $a(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$

1. A particle moves in the xy-plane so that at any time t, the position of the particle is given by $x(t) = t^3 + 4t^2$, $y(t) = t^4 - t^3$.
   a. Find the velocity vector when $t = 1$

$\frac{dx}{dt} = 3t^2 + 8t$ $\quad$ $\frac{dy}{dt} = 4t^3 - 3t^2$

$\left.\frac{dx}{dt}\right|_{t=1} = 3(1)^2 + 8(1) = 11$ $\quad$ $\left.\frac{dy}{dt}\right|_{t=1} = 4(1)^3 - 3(1)^2 = 1$

$\boxed{v(t) = \langle 11, 1 \rangle}$

   b. Find the acceleration vector when $t = 2$

$x''(t) = 6t + 8$ $\quad$ $y''(t) = 12t^2 - 6t$

$x''(2) = 20$ $\quad$ $y''(2) = 36$

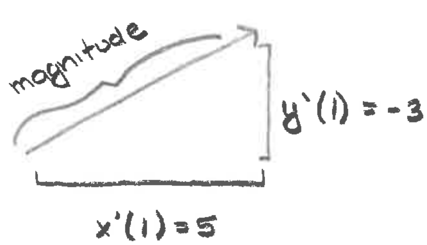
$\boxed{a(t) = \langle 20, 36 \rangle}$

2. A particle moves in the xy-plane so that at any time t, $t \geq 0$, the position of the particle is given by $x(t) = t^2 + 3t$, $y(t) = t^3 - 3t^2$. Find the magnitude of the velocity vector when $t = 1$.

speed

$x'(t) = 2t + 3$ $\quad$ $y'(t) = 3t^2 - 6t$

$x'(1) = 5$ $\quad$ $y'(1) = -3$



speed $= \sqrt{(5)^2 + (-3)^2}$

$\boxed{= \sqrt{34}}$

3. A particle moves in the xy-plane so that $x = \sqrt{3} - 4\cos t$ and $y = 1 - 2\sin t$, where $0 \le t \le 2\pi$. The path of the particle intersects the $x-$axis twice. Write an expression that represents the distance traveled by the particle between the two x-intercepts.

$$d = \int_{t_1}^{t_2} \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$$

$y = 0$

$1 - 2\sin t = 0$

$\sin t = 1/2$

$t = \pi/6,\ 5\pi/6$

$\frac{dx}{dt} = 4\sin t \qquad \frac{dy}{dt} = -2\cos t$

$$\boxed{d = \int_{\pi/6}^{5\pi/6} \sqrt{(4\sin t)^2 + (-2\cos t)^2}\, dt}$$

4. A particle moves in the xy-plane so that at any time t, the position of the particle is given by $x(t) = 2t^3 - 15t^2 + 36t + 5$, $y(t) = t^3 - 3t^2 + 1$, where $t \ge 0$. For what value(s) of t is the particle <u>at rest?</u>

$x'(t) = 0$ and $y'(t) = 0$

$x'(t) = 6t^2 - 30t + 36$

$0 = 6t^2 - 30t + 36$

$0 = 6(t^2 - 5t + 6)$

$0 = (t-3)(t-2)$

$t = 2, 3$

$y'(t) = 3t^2 - 6t$

$0 = 3t(t-2)$

$t = 0, 2$

$\boxed{\text{The particle is at rest at } t = 2}$

5. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 3t^2 - 4t, 8t^3 + 5 \rangle$. If the position vector at $t = 0$ is $\langle 7, -4 \rangle$, find the position of the particle at $t = 1$.

$x(0) = 7$ $\quad y(0) = -4$

$\frac{dx}{dt} = 3t^2 - 4t$ $\quad \frac{dy}{dt} = 8t^3 + 5$

$x(t) = t^3 - 2t^2 + C$ $\quad y(t) = 2t^4 + 5t + C$

$x(0) = 7 = 0^3 - 2(0)^2 + C$ $\quad y(0) = -4 = 2(0)^4 + 5(0) + C$

$7 = C$ $\quad -4 = C$

$x(t) = t^3 - 2t^2 + 7$ $\quad y(t) = 2t^4 + 5t - 4$

$t = 1$

$x(1) = 1^3 - 2(1)^2 + 7$
$= 6$

$y(1) = 2(1)^4 + 5(1) - 4$
$= 3$

$\boxed{r(2) = \langle 6, 3 \rangle}$

6. An object moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dx}{dt} = \sin(t^3)$ and $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the object is at position $(1, 4)$.

a. Find the acceleration vector for the particle at $t = 2$.

$x''(t) = 3t^2 \cos(t^3)$ $\quad y''(t) = -2t \sin(t^2)$

$x''(2) = 3(2)^2 \cos(2^3)$ $\quad y''(2) = -4 \sin(4)$

$= 12 \cos 8$

$a(2) = \langle -1.746, 3.027 \rangle$

b. Write the equation of the tangent line to the curve at the point where $t = 2$.

$\frac{dy}{dx} = \frac{\cos(t^2)}{\sin(t^3)}$

$\boxed{y - 4 = -0.661(x - 1)}$

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\cos 4}{\sin 8}$

$= -0.661$

c. Find the speed of the vector at $t = 2$.

$$\text{speed} = \sqrt{(\sin 8)^2 + (\cos 4)^2}$$

$$\boxed{= 1.186}$$

d. Find the position of the particle at time $t = 1$

* at $t=2$ given position $(1, 4)$

$x'(t) = \sin(t^3)$

$x(2) = x(1) + \int_1^2 x'(t)\,dt$

$1 = x(1) + \int_1^2 x'(t)\,dt$

$x(1) = 1 - \int_1^2 x'(t)\,dt$

$x(1) = 0.782$

$y'(t) = \cos(t^2)$

$y(2) = y(1) + \int_1^2 y'(t)\,dt$

$y(1) = 4 - \int_1^2 y'(t)\,dt$

$y(1) = 4.443$

$$\boxed{r(1) = \langle 0.782, 4.443 \rangle}$$