Velocity, Speed, Acceleration, and Direction of Motion
Suppose a particle moves along a smooth curve in the plane so that its position at any time t is $(x(t), y(t))$, where x and y are differentiable functions of t.

1. The particle's **position vector** is $r(t) = \langle x(t), y(t) \rangle$
2. The particle's **velocity vector** is $v(t) = \langle \frac{dx}{dt}, \frac{dy}{dt} \rangle$
3. The particle's **speed** is the magnitude of **v,** denoted $|v|$. Speed is a scalar not a vector.
4. The particle's **acceleration vector** is $a(t) = \langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$

1. A particle moves in the xy-plane so that at any time t, the position of the particle is given by $x(t) = t^3 + 4t^2$, $y(t) = t^4 - t^3$.
   a. Find the velocity vector when $t = 1$

   b. Find the acceleration vector when $t = 2$

2. A particle moves in the xy-plane so that at any time t, $t \geq 0$, the position of the particle is given by $x(t) = t^2 + 3t$, $y(t) = t^3 - 3t^2$. Find the magnitude of the velocity vector when $t = 1$.

3. A particle moves in the xy-plane so that $x = \sqrt{3} - 4\cos t$ and $y = 1 - 2\sin t,$ where $0 \le t \le 2\pi.$ The path of the particle intersects the $x-$axis twice. Write an expression that represents the distance traveled by the particle between the two x-intercepts.

4. A particle moves in the xy-plane so that at any time $t,$ the position of the particle is given by $x(t) = 2t^3 - 15t^2 + 36t + 5,$ $y(t) = t^3 - 3t^2 + 1,$ where $t \ge 0.$ For what value(s) of t is the particle at rest?

5. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 3t^2 - 4t, 8t^3 + 5 \rangle$. If the position vector at $t = 0$ is $\langle 7, -4 \rangle$, find the position of the particle at $t = 1$.

6. An object moving along a curve in the xy-plane has position $\langle x(t), y(t) \rangle$ at time t with $\frac{dx}{dt} = \sin(t^3)$ and $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the object is at position $(1, 4)$.
   a. Find the acceleration vector for the particle at $t = 2$.

   b. Write the equation of the tangent line to the curve at the point where $t = 2$.

c. Find the speed of the vector at $t = 2$.

d. Find the position of the particle at time $t = 1$