Want to use numerical integration to find areas enclosed by polar curves. It is not reasonable to convert all polar functions into rectangular equations. So we need to have a formula involving small changes in θ rather than x.

A small change Δx produces a thin rectangular strip of area

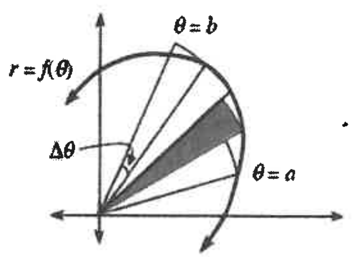
A small change $\Delta\theta$ produces a thing circular sector of area

$\frac{\text{sector}}{\pi r^2} = \frac{\theta}{360^\circ}$

$\text{sector} = \theta \frac{\pi r^2}{360}$

$= \frac{1}{2} r^2$

★ $\frac{\pi}{180}$ convert to radians



Area of a sector: $\frac{1}{2}r^2\theta$

Area of highlighted sector: $\frac{1}{2} f(\theta)^2 \,\Delta\theta$

Calculate entire enclosed area: $\frac{1}{2}\int_{\theta_1}^{\theta_2} f(\theta)^2 d\theta$

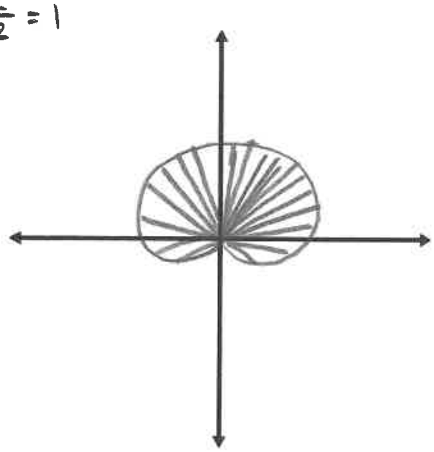
Area in Polar Coordinates

The area of the region between the original and the curve $r = f(\theta)$ for $\alpha \le \theta \le \beta$ is:

$$A = \int_{\alpha}^{\beta} \tfrac{1}{2} f(\theta)^2 \, d\theta$$

1. Find the area bounded by the graph of $r = 2 + 2\sin\theta$

$\frac{2}{2} = 1$



$$A = \int_0^{2\pi} \frac{1}{2}(2 + 2\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (4 + 8\sin\theta + 4\sin^2\theta)\, d\theta$$

$$= \int_0^{2\pi} (2 + 4\sin\theta + 2\sin^2\theta)\, d\theta$$

★ $\cos 2\theta = 1 - 2\sin^2\theta$

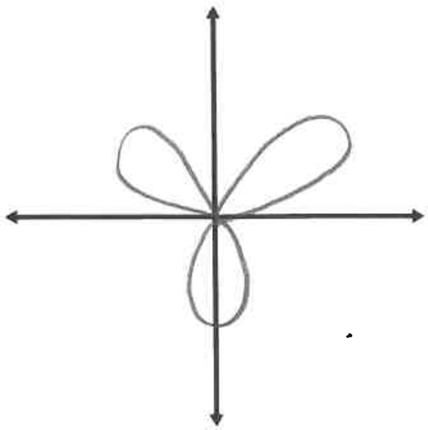
$$= \int_0^{2\pi} (2 + 4\sin\theta + 1 - \cos 2\theta)\, d\theta$$

$$= 3\theta - 4\cos\theta - \frac{1}{2}\sin 2\theta \Big|_0^{2\pi}$$

$$= 3(2\pi) - 4\cos 2\pi - \frac{1}{2}\sin 4\pi - \left(0 - 4\cos 0 - \frac{1}{2}\sin 0\right)$$

$$= 6\pi - 4 + 4 \quad = \boxed{6\pi}$$

2. Find the area of one petal of $r = 2\sin 3\theta$



$\sin 3\theta = 1$ and -1

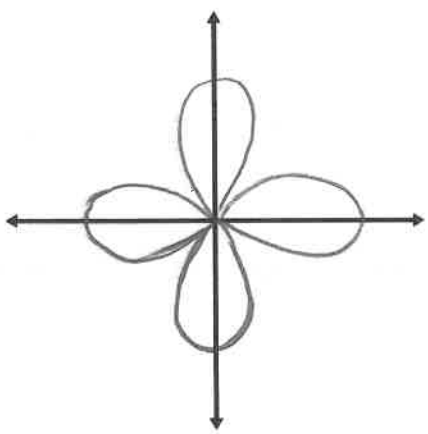
$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}$

$A = \frac{1}{2}\int_0^{\pi/3} (2\sin 3\theta)^2 \, d\theta$

$= \boxed{\frac{\pi}{3}}$

3. Find the area of one petal of $r = 4\cos 2\theta$



$\cos 2\theta = 0$

$2\theta = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

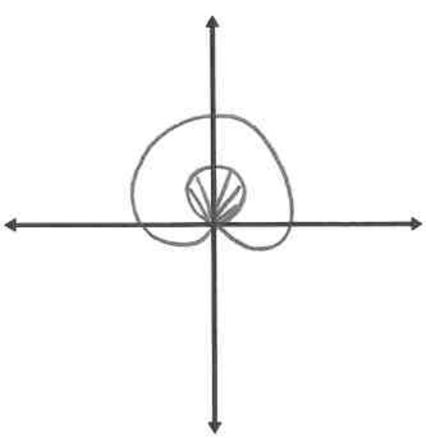
$\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$A = \int_{\pi/4}^{3\pi/4} \frac{1}{2}(4\cos 2\theta)^2 \, d\theta$

$= \boxed{2\pi}$

* look @ desmos trace

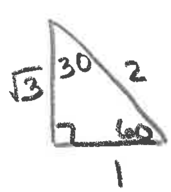
4. Find the area of the inner loop of the graph of $r = 1 + 2\sin\theta$



$1 + 2\sin\theta = 0$

$\sin\theta = -1/2$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



S | A
T | C

$A = \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)^2 \, d\theta$

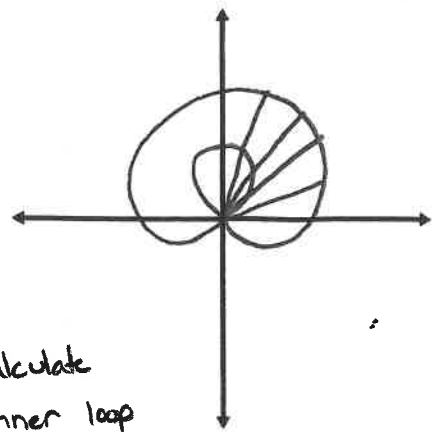
$= \frac{2\pi - 3\sqrt{3}}{2}$

$\approx \boxed{0.544}$

5. Find the area of the graph $r = 1 + 2\sin\theta$

*counts inner piece

outer → $-\pi/6$ to $7\pi/6$



① $A = \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2\, d\theta$

$= \boxed{8.881}$

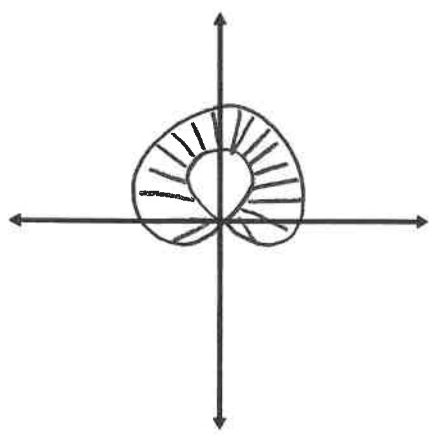
OR

② $A = \frac{1}{2}\int_{0}^{2\pi} (1+2\sin\theta)^2\, d\theta - \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\, d\theta$

$= 8.881$

*when calculate sector, inner loop gets included

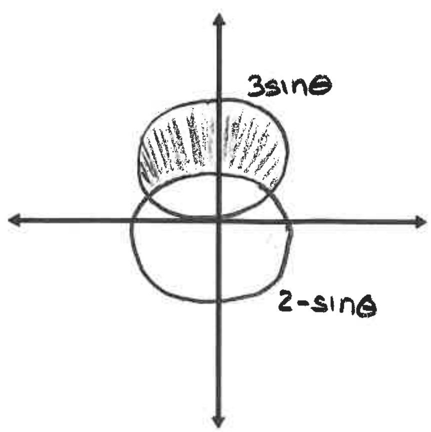
6. Find the area between the two loops for the graph of $r = 1 + 2\sin\theta$



$A = \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1+2\sin\theta)^2\, d\theta - \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\, d\theta$

$= \boxed{8.338}$

7. Find the area inside of $r = 3\sin\theta$ and outside $r = 2 - \sin\theta$



$3\sin\theta = 2 - \sin\theta$

$4\sin\theta = 2$

$\sin\theta = 1/2$

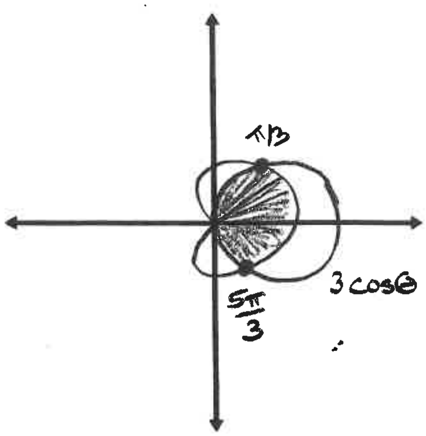
$\theta = \pi/6,\ 5\pi/6$

$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left[(3\sin\theta)^2 - (2-\sin\theta)^2\right] d\theta$

*square each individually like would for washer

$= \boxed{5.196}$

8. Find the area of the common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$



$3\cos\theta = 1 + \cos\theta$

$2\cos\theta = 1$

$\cos\theta = 1/2$

$\theta = \pi/3,\ 5\pi/3$

not $5\pi/3$ b/c on 2nd revolution @ $5\pi/3$

$$A = \frac{1}{2}\int_{-\pi/3}^{\pi/3} (1+\cos\theta)^2\, d\theta + \frac{1}{2}\int_{\pi/3}^{2\pi/3} (3\cos\theta)^2\, d\theta$$

★ Remember

3 cosθ gets full shape from $[0, \pi]$

★ $\cos(2\pi/3) = -1/2$ so lands in same spot as $\cos(5\pi/3)$

OR

$$A = 2\left(\frac{1}{2}\int_{0}^{\pi/3} (1+\cos\theta)^2\, d\theta\right) + 2\left(\frac{1}{2}\int_{\pi/3}^{\pi/2} (3\cos\theta)^2\, d\theta\right)$$

$$= \boxed{\frac{5\pi}{4}}$$