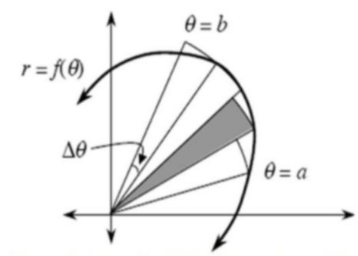
Want to use numerical integration to find areas enclosed by polar curves. It is not reasonable to convert all polar functions into rectangular equations. So we need to have a formula involving small changes in θ rather than x.

A small change Δx produces a thin rectangular strip of area

A small change $\Delta\theta$ produces a thing circular sector of area



Area of a sector: $\frac{1}{2}r^2\theta$

Area of highlighted sector: $\frac{1}{2}f(\theta)^2\ \Delta\theta$

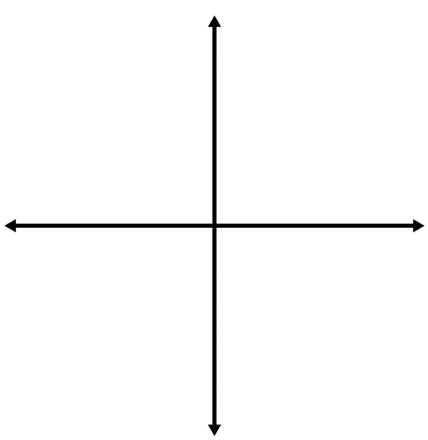
Calculate entire enclosed area: $\frac{1}{2}\int_{\theta_1}^{\theta_2} f(\theta)^2 d\theta$

Area in Polar Coordinates

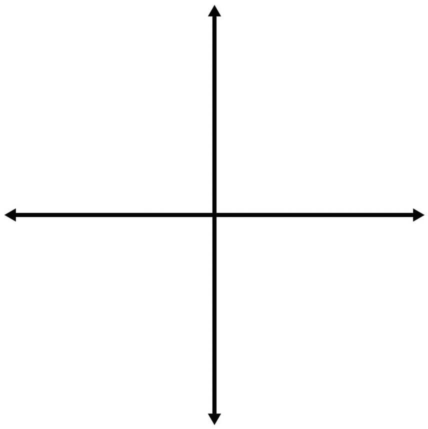
The area of the region between the original and the curve $r = f(\theta)$ for $\alpha \le \theta \le \beta$ is:

$$A = \int_{\alpha}^{\beta} \tfrac{1}{2}f(\theta)^2\ d\theta$$

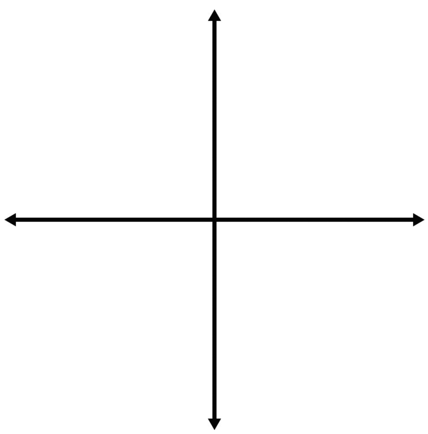
1. Find the area bounded by the graph of $r = 2 + 2\sin\theta$



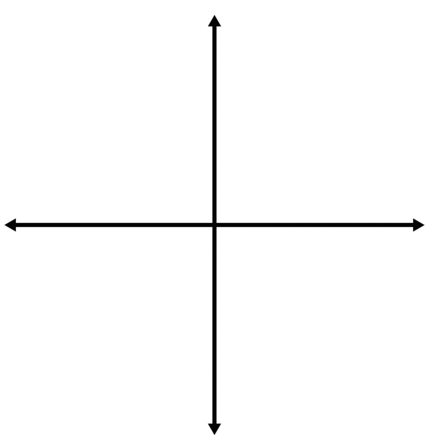
2. Find the area of one petal of $r = 2\sin 3\theta$



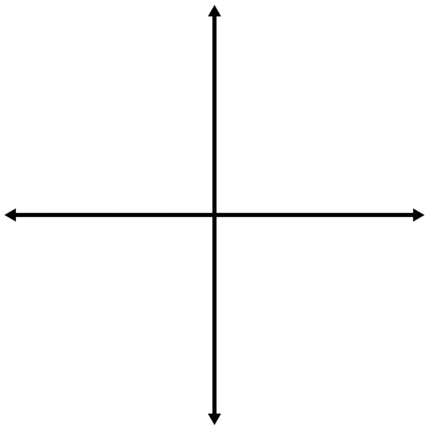
3. Find the area of one petal of $r = 4\cos 2\theta$



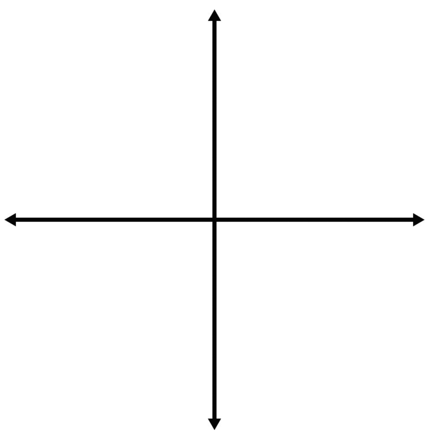
4. Find the area of the inner loop of the graph of $r = 1 + 2\sin\theta$



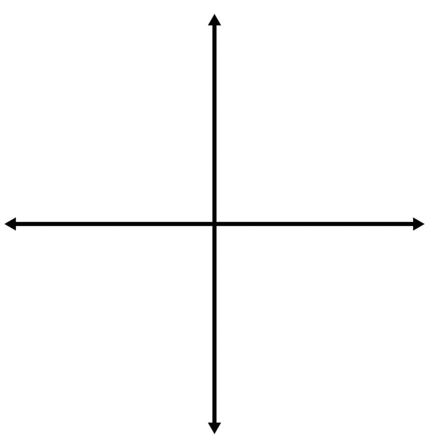
5. Find the area of the graph $r = 1 + 2\sin\theta$



6. Find the area between the two loops for the graph of $r = 1 + 2\sin\theta$



7. Find the area inside of $r = 3\sin\theta$ and outside $r = 2 - \sin\theta$



8. Find the area of the common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$