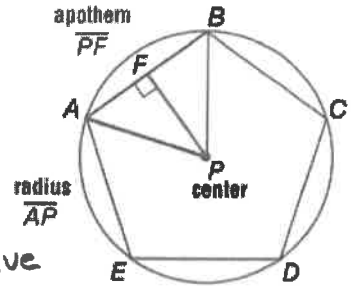
Apothem: a segment drawn from the center of a regular polygon perpendicular to a side of the polygon

Central Angle of a Regular Polygon: has its vertex at the center of the polygon and its sides pass through consecutive vertices of the polygon

$\frac{360}{n}$ $n \rightarrow$ # of sides



$\angle APB$ is a central angle of regular pentagon $ABCDE$.

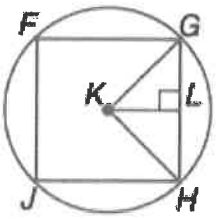
1. Identify the following:

Radius: $\overline{HG}$ or $\overline{HH}$ Central Angle: $\angle GHH$

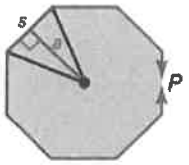
Diameter: none labeled

Measure of the Central Angle $= \frac{360}{4} = 90°$

Apothem: $\overline{HL}$



2. Area of a Regular Polygon:



A = (area of one $\triangle$) $\cdot$ (number of triangles)

$= \frac{1}{2}$(base)(height) $\cdot$ (number of triangles)

$= \frac{1}{2} \cdot s \cdot a \cdot n$

$= \frac{1}{2} a \cdot (sn) = \frac{1}{2} aP$

* $s \cdot n$
s perimeter *

Area of a Regular Polygon: The area of a regular n-gon with side length s is half the product of the apothem *a* and the perimeter P, so

$$A = \frac{1}{2}aP \text{ or } A = \frac{1}{2}a * ns$$

a = apothem
P = perimeter
$= n \cdot s$
n = # of sides
s = side length

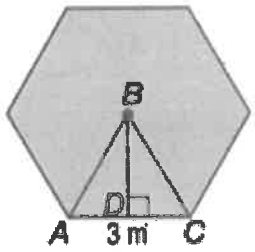
Steps (finding area of a regular polygon):

1. Central Angle
2. Solve the triangle formed with the apothem
   a. Solve all angles
   b. Solve apothem
   c. Solve side
3. Find the perimeter
4. Find area

3. Find the area of each regular polygon. Round your answer to the nearest tenth:

a.

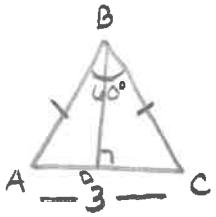
6 sides

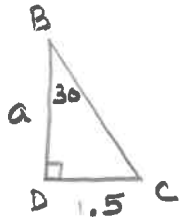


① Central Angle

$\frac{360}{6} = 60°$

② Triangle





b) apothem → $\tan 30 = \frac{1.5}{a}$

$a \tan 30 = 1.5$

$a = \frac{1.5}{\tan 30}$

$a \approx 2.59$

c) side = 3

③ Perimeter

$P = n \cdot s$

$= 6 \cdot 3$

$= 18$

Area = $\frac{1}{2} a P$

$= \frac{1}{2}(2.59)(18)$

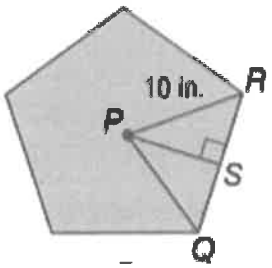
$\approx 23.3 \text{ m}^2$

b.

apothem ≈ 8.09

side = 11.74

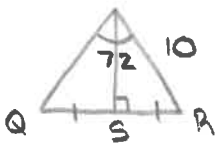
n = 5

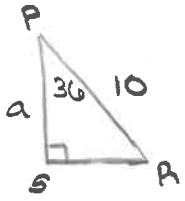


① Central Angle

$\frac{360}{5} = 72°$

② Triangle





apothem

$\cos 36 = \frac{a}{10}$

$10 \cos 36 = a$

$8.09 \approx a$

side

$\sin 36 = \frac{SR}{10}$

$10 \sin 36 = SR$

$5.87 \approx SR$

side = 2(5.87)

≈ 11.74

③ Perimeter

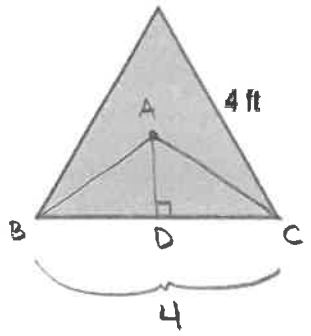
$P = 5(11.74)$

$= 58.7$

Area = $\frac{1}{2}(8.09)(58.7)$

$= 237.4 \text{ in}^2$

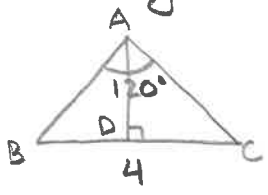
c.

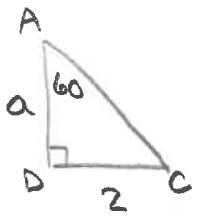


① central angle

$\frac{360}{3} = 120°$

② Triangle





apothem

$\tan 60 = \frac{2}{a}$

$a \tan 60 = 2$

$a = \frac{2}{\tan 60}$

$a \approx 1.15$

apothem ≈ 1.15

side $= 4$

$n = 3$

③ Perimeter

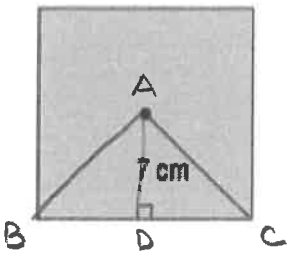
$P = 3 \cdot 4$

$= 12$

Area $= \frac{1}{2}(1.15)(12)$

$= 6.9 \text{ ft}^2$

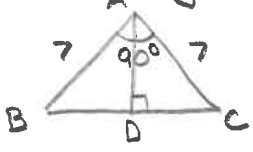
d.

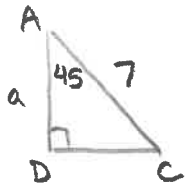


① central Angle

$\frac{360}{4} = 90°$

② Triangle





apothem

$\cos 45 = \frac{a}{7}$

$7 \cos 45 = a$

$4.94 \approx a$

side

$\sin 45 = \frac{DC}{7}$

$7 \sin 45 = DC$

$4.94 \approx DC$

apothem ≈ 4.94

side $= 2(4.94) = 9.88$

$n = 4$

③ Perimeter

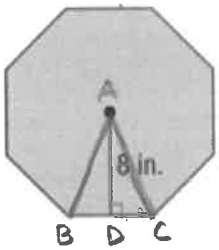
$P = 4(9.88)$

$= 39.52$

$A = \frac{1}{2}(4.94)(39.52)$

$= 97.6 \text{ cm}^2$

e.



apothem = 8

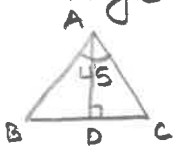
side = 2(3.31) = 6.62

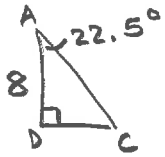
n = 8

① central Angle

$\frac{360}{8} = 45°$

② Triangle





side

$\tan 22.5 = \frac{DC}{8}$

$8\tan 22.5 = DC$

$3.31 \approx DC$

③ Perimeter

$P = 8(6.62)$

$= 52.96$

$A = \frac{1}{2}(8)(52.96)$

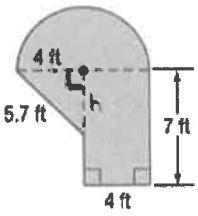
$= 211.84\ in^2$

Composite Figure:

a figure that can be separated into regions that are basic figures, such as triangles, rectangles, trapezoids, and circles

4. Find the area of each composite figure:

a.



A = semi-circle + triangle + rectangle

$= \frac{1}{2}\pi(4)^2 + \frac{1}{2}$ base · height $+ 4(7)$

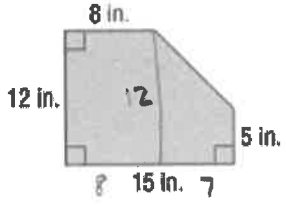
$= \frac{1}{2}\pi(4)^2 + \frac{1}{2}(4)(4.06) + 4(7)$

$= 61.3\ ft^2$

$4^2 + h^2 = 5.7^2$

$h = 4.06$

b.

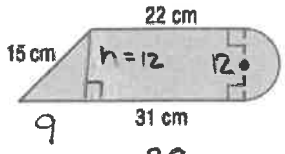


Area = rectangle + trapezoid

$= 8(12) + \frac{1}{2}(7)(5+12)$

$= 155.5\ in^2$

c.



$15^2 = 9^2 + h^2$

$225 = 81 + h^2$

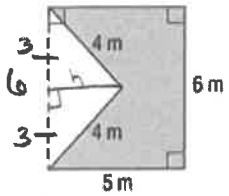
$12 = h$

Area = trapezoid + semicircle

$= \frac{1}{2}(12)(22+31) + \frac{1}{2}\pi(6)^2$

$= 374.5\ cm^2$

d.



$3^2 + h^2 = 4^2$

$9 + h^2 = 16$

$h^2 = 7$

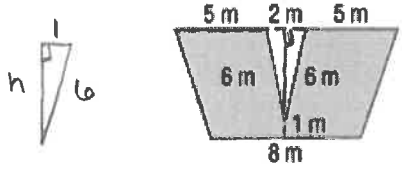
$h = \sqrt{7}$

Area = rectangle − triangle

$= 5(6) - \frac{1}{2}(6)(\sqrt{7})$

$= 22.1\ in^2$

e.



$1^2 + h^2 = 6^2$

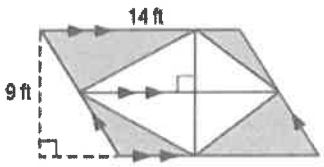
$h = \sqrt{35}$

Area = trapezoid − triangle

$= \frac{1}{2}(\sqrt{35}+1)(12+8) - \frac{1}{2}(2)(\sqrt{35})$

$= 63.2\ m^2$

f.



Area = parallelogram − kite

$= (14)(9) - \frac{1}{2}(14)(9)$

$= 63\ ft^2$