**Theorem** Let  $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ , where f, g, and h are differentiable

functions, then 
$$\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$$

Note: If  $\mathbf{r}(t)$  is a **position function** of a particle at time t, then the **velocity function** is  $\mathbf{r}'(t) = \mathbf{v}(t)$  and the **acceleration function** is  $\mathbf{r}''(t) = \mathbf{v}'(t) = \mathbf{a}(t)$ .

**Definition:** The unit tangent vector at t is defined to be  $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$ 

**Theorem** Suppose  $\mathbf{u}$  and  $\mathbf{v}$  are differentiable vector functions, c is a scalar, and f is a real-valued function. Then.

$$\begin{split} \frac{d}{dt}[\mathbf{u}(t)+\mathbf{v}(t)] &= \mathbf{u}'(t)+\mathbf{v}'(t) \\ \frac{d}{dt}[c\mathbf{u}(t)] &= c\mathbf{u}'(t) \\ \frac{d}{dt}[c\mathbf{u}(t)] &= c\mathbf{u}'(t) \\ \frac{d}{dt}[f(t)\mathbf{u}(t)] &= f'(t)\mathbf{u}(t)+f(t)\mathbf{u}'(t) \\ \end{split}$$

$$\frac{d}{dt}[\mathbf{u}(t)\times\mathbf{v}(t)] &= \mathbf{u}'(t)\times\mathbf{v}(t)+\mathbf{u}(t)\times\mathbf{v}'(t) \\ \frac{d}{dt}[f(t)\mathbf{u}(t)] &= f'(t)\mathbf{u}'(t) \\ \frac{d}{dt}[\mathbf{u}(t)\times\mathbf{v}(t)] &= f'(t)\mathbf{u}'(t) \\ \end{split}$$
chain rule

Example: Given  $\mathbf{r}(t) = \langle 3t, e^{2t-4}, \sin(t\pi) \rangle$ .

(a) Find a tangent vector to the curve at t = 0.

(b) Find  $\mathbf{T}(0)$ .

(c) Find a tangent line to the curve at the point (6,1,0)

## 1.7 Derivative and Integrals of Vector Functions Multivariable

Example: Given  $\mathbf{r}(t)=\langle 3t,\ e^{2t-4},\ \sin(t\pi)\rangle.$  compute  $\int \mathbf{r}(t)\ dt$ 

Example: The function  ${\bf r}(t)=\langle t-\sin(t),1-\cos(t)\rangle$  traces a cycloid. Find the points where:

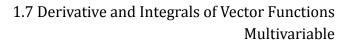
a.  $\mathbf{r}'(t)$  is horizontal and nonzero.

b.  $\mathbf{r}'(t)$  is the zero vector

## 1.7 Derivative and Integrals of Vector Functions Multivariable

Let's prove that if  $\mathbf{r}(t)$  has constant length (i.e.  $|\mathbf{r}(t)| = c$  (some constant)), then  $\mathbf{r}(t)$  is orthogonal to  $\mathbf{r}'(t)$ .

Example: Evaluate 
$$\int_{-1}^{3} \langle 8t^2 - t, 6t^3 + t \rangle dt$$



Example: Find the location and velocity at t = 4 of a particle whose path satisfies

$$\frac{d\mathbf{r}}{dt} = \langle 2t^{-1/2}, 6, 8t \rangle \qquad \mathbf{r}(1) = \langle 4, 9, 2 \rangle$$

Example: Sketch the curve parametrized by  $\mathbf{r}(t) = \langle 1 - t^2, t \rangle$  for  $-1 \le t \le 1$ . Compute the tangent vector at t = 1 and add it to the sketch.