**Theorem** Let $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, where $f$, $g$, and $h$ are differentiable functions, then $\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t)\rangle = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$

Note: If $\mathbf{r}(t)$ is a **position function** of a particle at time $t$, then the **velocity function** is $\mathbf{r}'(t) = \mathbf{v}(t)$ and the **acceleration function** is $\mathbf{r}''(t) = \mathbf{v}'(t) = \mathbf{a}(t)$.

**Definition:** The **unit tangent vector** at $t$ is defined to be $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

**Theorem** Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, $c$ is a scalar, and $f$ is a real-valued function. Then.

$$\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t) \qquad \frac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$$

$$\frac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t) \qquad \frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$$

$$\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t) \qquad \frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t)) \quad \text{chain rule}$$

Example: Given $\mathbf{r}(t) = \langle 3t,\ e^{2t-4},\ \sin(t\pi)\rangle$.

(a) Find a tangent vector to the curve at $t = 0$.

(b) Find $\mathbf{T}(0)$.

(c) Find a tangent line to the curve at the point $(6, 1, 0)$

Example: Given $\mathbf{r}(t) = \langle 3t,\ e^{2t-4},\ \sin(t\pi)\rangle$. compute $\int \mathbf{r}(t)\ dt$

Example: The function $\mathbf{r}(t) = \langle t - \sin(t), 1 - \cos(t)\rangle$ traces a cycloid. Find the points where:

a. $\mathbf{r}'(t)$ is horizontal and nonzero.

b. $\mathbf{r}'(t)$ is the zero vector

Let's prove that if $\mathbf{r}(t)$ has constant length (i.e. $|\mathbf{r}(t)| = c$ (some constant)), then $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$.

Example: Evaluate $\int_{-1}^{3} \langle 8t^2 - t, 6t^3 + t \rangle dt$

Example: Find the location and velocity at $t = 4$ of a particle whose path satisfies

$$\frac{d\mathbf{r}}{dt} = \langle 2t^{-1/2}, 6, 8t \rangle \qquad \mathbf{r}(1) = \langle 4, 9, 2 \rangle$$

Example: Sketch the curve parametrized by $\mathbf{r}(t) = \langle 1 - t^2, t \rangle$ for $-1 \leq t \leq 1$. Compute the tangent vector at $t = 1$ and add it to the sketch.