In Calculus 2, the length of a two-dimensional smooth curve that is only traversed once on an interval I was given by:

This can be extended to a space curve. If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ on the interval $a \leq t \leq b$, then the length of the curve is given by:

A curve $\mathbf{r}(t)$ is called smooth on an interval if $\mathbf{r}'(t)$ is continuous and $\mathbf{r}'(t) \neq 0$ on the interval. A smooth curve has no sharp corners or cusps → the tangent vector has continuous movement.

1. Find the length of the arc for $\mathbf{r}(t) = \langle 3t, 2\sin(t), 2\cos(t) \rangle$ from the point (0, 0, 2) to (6π, 0, 2).

Arc Length Function, s, is

$$s(t) = \int_a^t |\mathbf{r}'(u)| du$$

The arc length s is called the arc length parameter

Speed at time $t = \frac{ds}{dt} = |\mathbf{r}'(t)|$

2. Find the arc length function for $\mathbf{r}(t) = \langle e^t, e^t \sin(t), e^t \cos(t) \rangle$ from the point $(1, 0, 1)$ in the direction of increasing t.

3. Reparametrize the curve $\mathbf{r}(t) = \langle 1 + 2t, 3 + t, -5t \rangle$ with respect to arc length measured from the point where $t = 0$ in the direction of increasing t.

4. Find the arc length parametrization of the helix $\mathbf{r}(t) = \langle \cos(4t), \sin(4t), 3t \rangle$