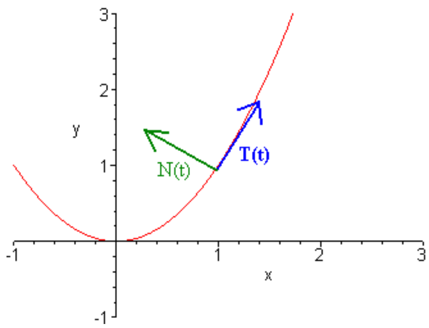
Normal Vector
In 14.2 we proved $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$, which can be extended to $\mathbf{T}'(t)$ and $\mathbf{T}(t)$. The unit vector in the direction of $\mathbf{T}'(t)$ is called the **normal vector** and denoted $\mathbf{N}(t)$.

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{||\mathbf{T}'(t)||}$$

or

$$\mathbf{T}'(t) = v(t)\kappa(t)\mathbf{N}(t)$$

For a plane curve, $\mathbf{N}(t)$ points in the direction of bending



1. Consider the helix

$$\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), t\rangle$$

Show that for all t, the normal vector is parallel to the xy −plane and points toward the z-axis.

Binormal Vector

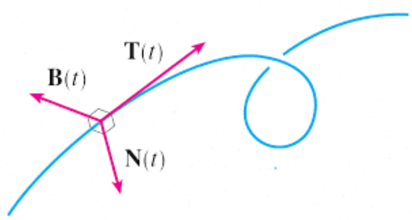
At a point *P* on a curve, the vectors **T** and **N** determine a plane. The normal vector to this plane, **B (Binormal Vector)** is defined by

$$\mathbf{B} = \mathbf{T} \times \mathbf{N}$$

B is orthogonal to both **T** and **N**

B is a unit vector since $||\mathbf{B}|| = ||\mathbf{T}||\ ||\mathbf{N}|| \sin(\pi/2) = (1)(1)(1) = 1$

Frenet Frame the set of mutually perpendicular vectors **B, T,** and **N.**



2. Determine the binormal vector for the helix

$$\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), t\rangle$$

3. Find the normal and binormal vectors for

$$\vec{r}\,(t) = \langle t, 3\sin t, 3\cos t\rangle.$$