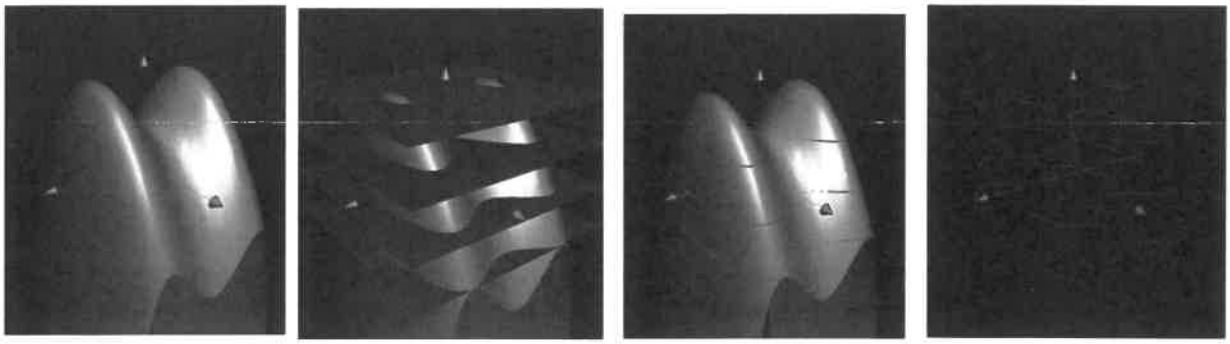
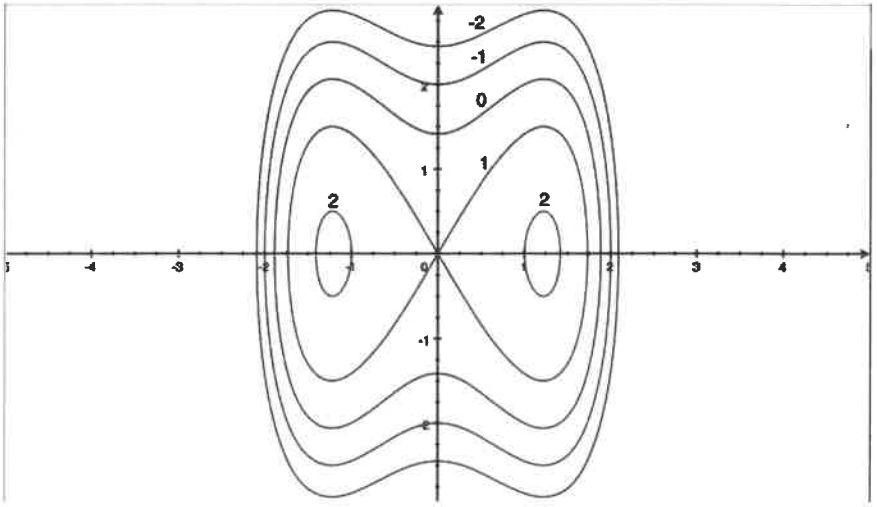
Let $z = f(x,y)$ be a function whose graph is a surface in R^3 . Suppose this graph is intersected by a plane $z = k$, parallel to the xy-plane. This is equivalent to holding z constant and reducing the equation into an implicit function of x and y only—i.e. written $f(x,y) = k$





Note: The choice of outputs you want to represent, such as $\{-2,-1,0,1,2\}$ in this example, should almost always be evenly spaced. This makes it much easier to understand the "shape" of the function just by looking at the contour map.

1. Sketch level curves (contour map) for $f(x,y) = x^2 + y^2$

$z = x^2 + y^2$

let $z = 1$ $\quad 1 = x^2 + y^2$

$z = 2$ $\quad 2 = x^2 + y^2$

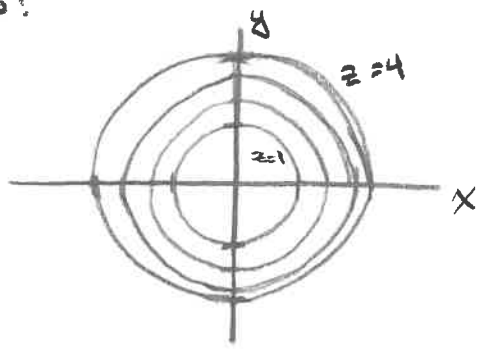
$z = 3$ $\quad 3 = x^2 + y^2$

$z = 4$ $\quad 4 = x^2 + y^2$

circles!

* can recognize shape

$z = x^2 + y^2$



2. Sketch a contour graph for $f(x,y) = \ln(y^2 - x)$

$z = \ln(y^2 - x)$

let $z = 0$ $\quad 0 = \ln(y^2 - x)$

$e^0 = y^2 - x$

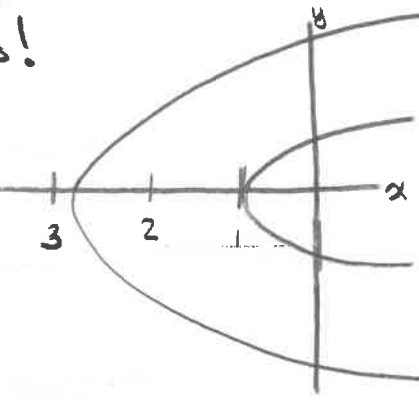
$1 = y^2 - x$

$1 - y^2 = x$

$z = 1$ $\quad e^1 - y^2 = x$

$z = 2$ $\quad e^2 - y^2 = x$

parabolas!



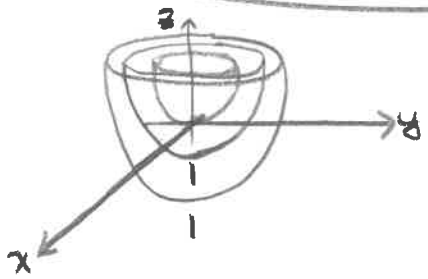
3. Sketch the level surfaces for $f(x,y,z) = x^2 + y^2 - z$

$w = x^2 + y^2 - z$

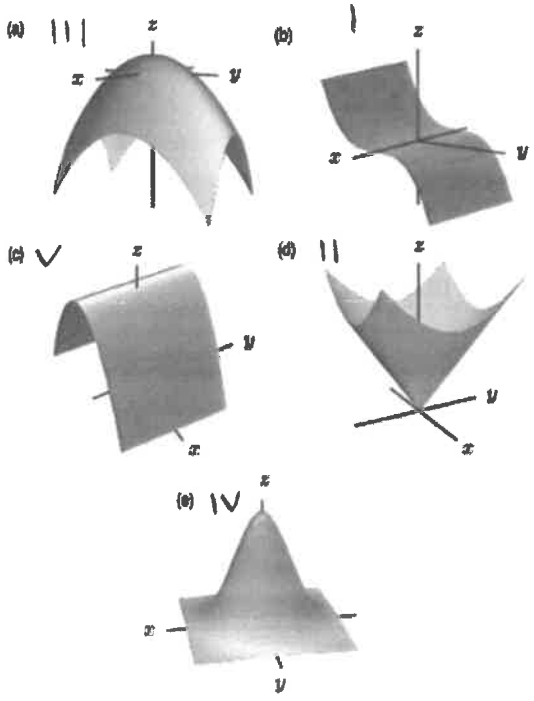
let $w = 0$ $\quad z = x^2 + y^2$

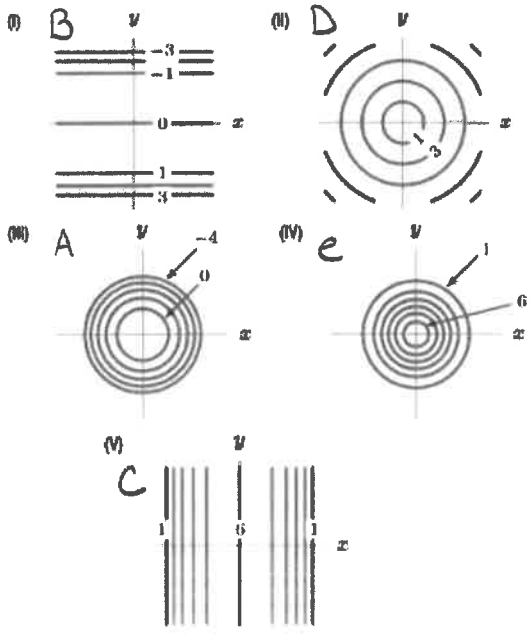
$w = 1$ $\quad z = x^2 + y^2 - 1$

$w = 2$ $\quad z = x^2 + y^2 - 2$



Match the surfaces (a)-(e) in the figure below with their respective contour diagrams





Match the equations with their contours.

(i) B $z = \dfrac{1}{x^2+y^2+1}$ (ii) A $z = 2x^2 + y$ (iii) C $z = \dfrac{y}{x+1}$ (iv) D $z = \dfrac{1}{x^2-2y^2}$

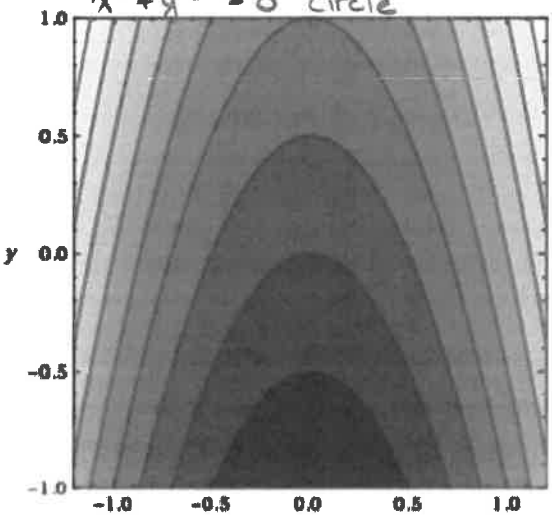
$2 = \frac{1}{x^2+y^2+1}$ $1 = \frac{1}{x^2+y^2+1}$ $x^2+y^2+1 = 1$ $x^2+y^2 = 0$ circle

$2x^2-2y^2-2=1$ no solution $-1 \le z \le 1$

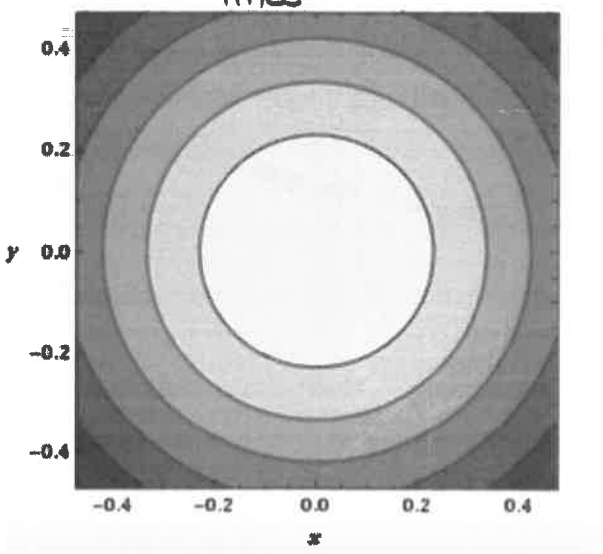
$0 = 2x^2+y$ $y = -2x^2$ parabolas!

$0 = \frac{y}{x+1}$ $0 = y$ $1 = \frac{y}{x+1}$ $x+1 = y$ lines

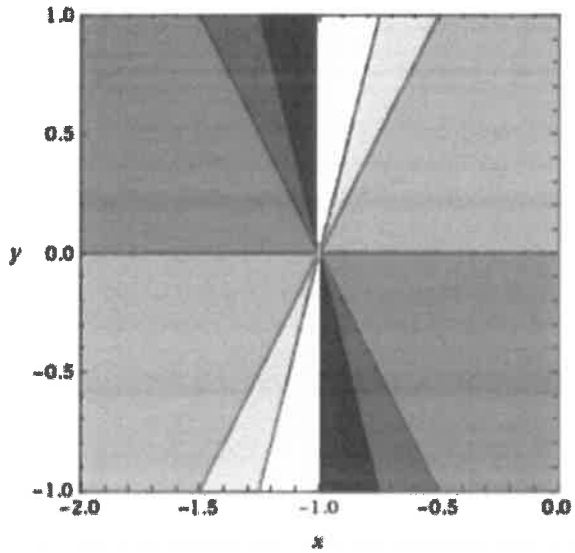
$1 = \frac{1}{x^2-2y^2}$ $x^2-2y^2=1$ $2 = \frac{1}{x^2-2y^2}$ $2x^2-2y^2=1$ hyperbolas!



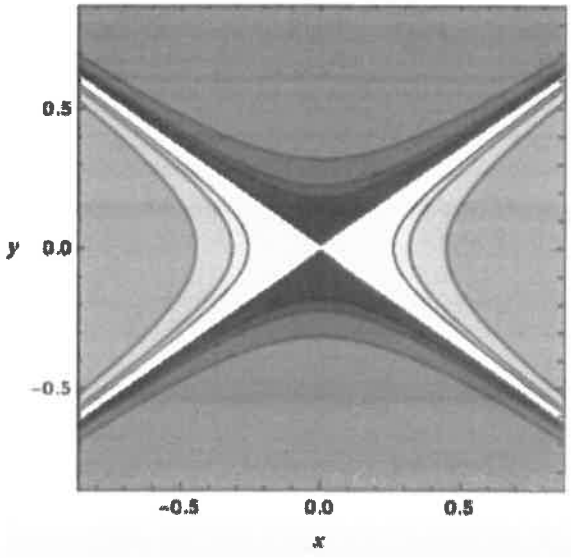
A (ii)



B (i)



C (iii)



D (iv)