Limit Laws Assume that $\lim_{(x,y)\to P} f(x,y)$ and $\lim_{(x,y)\to P} g(x,y)$ exist. Then:

(i) Sum Laws:

$$\lim_{(x,y)\to P} (f(x,y) + g(x,y)) = \lim_{(x,y)\to P} f(x,y) + \lim_{(x,y)\to P} g(x,y)$$

(ii) Constant Multiple Law:

$$\lim_{(x,y)\to P} kf(x,y) = k \lim_{(x,y)\to P} f(x,y)$$

(iii) Product Law:

$$\lim_{(x,y)\to P} f(x,y)g(x,y) = \left(\lim_{(x,y)\to P} f(x,y)\right)\left(\lim_{(x,y)\to P} g(x,y)\right)$$

(iv) Quotient Law:

$$\text{If } \lim_{(x,y)\to P} g(x,y) \neq 0, \text{ then}$$

$$\lim_{(x,y)\to P} \frac{f(x,y)}{g(x,y)} = \frac{\lim_{(x,y)\to P} f(x,y)}{\lim_{(x,y)\to P} g(x,y)}$$

Definition of Continuity

1. Show that $f(x,y) = \frac{3x+y}{x^2+y^2+1}$ is continuous and evaluate $\lim_{(x,y)\to(1,2)} f(x,y)$

2. Evaluate $\lim_{(x,y)\to(3,0)} x^3 \frac{\sin y}{y}$

A Composition of Continuous Functions is Continuous
If a function of two variables f is continuous at (a, b) and a function of one variable G is continuous at $c = f(a, b)$, then the composite function $G(f(x, y))$ is continuous at (a, b)

3. Write $H(x, y) = e^{-x^2+2y}$ as a composite function and evaluate $\lim_{(x,y)\to(1,2)} H(x, y)$

4. $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$